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ASYMMETRIC FOUR-DIMENSIONAL MANIFOLDS

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One of the most interesting questions in the theory of transformation groups was the question about the existence of manifolds which lack any symmetry i.e., there is no nontrivial periodic selfhomeomorphism on them. First examples of such manifolds were constructed by Conner, Raymond and Weinberger in [5]. The base for their construction was the following result due to A. Borel.

Let G be a finite group which acts on a closed connected manifold M. Suppose that there is a point $m \in M$ which is fixed by G. Then the group G acts on $\pi_1(M,m)$ and there is a homomorphism

$$\psi\colon G \to \operatorname{Aut}(\pi_1(M, m))$$

given by

$$\psi(g) = g_*\colon \pi_1(M, m) \to \pi_1(M, m), \quad \text{for } g \in G.$$

If there is no point of M which is fixed by G then one can still define (see [5]) a homomorphism

$$\varphi\colon G \to \operatorname{Out} \pi_1(M, *),$$

where

$$\operatorname{Out} \pi_1(M, *) := \operatorname{Aut} \pi_1(M, *)/\text{Inner automorphisms of } \pi_1(M, *).$$

Now the theorem of A. Borel states that in the case when M is aspherical (i.e., $\pi_i(M) = 0$ for all $i \geqslant 2$) then the homomorphism ψ is a monomorphism and if $\pi_1(M, *)$ is centerless then φ is a monomorphism as well. Therefore to find an example of a manifold with no nontrivial periodic selfhomeomorphism it is enough to be able to control the automorphisms of $\pi_1(M, *)$ for some aspherical manifold M. This was in fact the main idea in the construction of examples in [5]. Many other examples were constructed (see [1], [4], [12]) using the above idea (a different approach was presented by L. Siebenmann in [15]). In all these examples manifolds which were constructed were non simply-connected. One can say more, in fact the fundamental group was the main algebraic invariant which was used to decide that the constructed examples are indeed what we are looking for. The intriguing question, asked by P. Conner, F. Raymond, R. Schultz [see

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