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SPECIALIZATION OF CRYSTALLINE COHOMOLOGY

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Introduction. Let k be a perfect field of characteristic $p > 0$ and let $X \to S$ be a proper smooth morphism of k-schemes. The purpose of this paper is to prove, for the relative crystalline cohomology of the morphism $X \to S$, an analogue of Grothendieck's specialization theorem on F-crystals: the Newton polygons of the fibers are constant on the strata of a constructible stratification of S, and rise under specialization.

Whenever the relative crystalline cohomology groups $R^if_{\mathrm{cris}_*}\mathcal{O}_{X/W}$ are F-crystals on S/W ($W = W(k)$, the ring of Witt vectors of k) Grothendieck's original theorem ([6] Appendix or [8] 2.3) can be applied directly. Usually this is not the case, however, for the $\mathcal{O}_{S/W}$-modules $R^if_{\mathrm{cris}_*}\mathcal{O}_{X/W}$ are coherent but not always locally free; the most that the base-change theorems of crystalline cohomology guarantee is that $Rf_{\mathrm{cris}_*}\mathcal{O}_{X/W}$ is a "crystal in the derived category" ([2] 7.11). On the other hand it is often possible to get some results with ℓ-adic techniques—this can be done, for example, when S is a smooth curve and the fibers of $X \to S$ are projective, in which case one can combine the comparison theorem of Katz-Messing [9] with some results of Deligne ([5] 1.10.7) to deduce the specialization property, *provided* one already knows that the Newton polygons behave constructibly.

That it is at all possible to give a proof of the specialization theorem for a general quasicompact k-scheme S and proper smooth $X \to S$ is due to recent progress in the theory of F-isocrystals, namely the construction and study due to Ogus, of the category of *convergent F-isocrystals* [10]. In §1 we recall, for the reader's convenience, some aspects of this theory. In §2 we show that a convergent F-isocrystal satisfies an analogue of the specialization theorem, and use it to deduce the corresponding result in crystalline cohomology.

There are some other invariants of a crystalline nature whose behavior under specialization is still unknown. For example nothing is known about the behavior of the *torsion* in the crystalline cohomology of the fibers, apart from the fact that it need not be locally constant. One can also consider the invariants $T^{i,j}$ describing the number of "unipotent factors" in the E_1 term of the slope spectral sequence ([7] Intro. 2.4.1). In [3] it was shown that in a smooth proper family $X \to S$ of relative dimension two, the invariant $T^{0,2}$ increases under specialization (in this case the other $T^{i,j}$ are all zero), but nothing is known about the higher-dimensional case.

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