Vol. 53, No. 2 DUKE MATHEMATICAL JOURNAL © June 1986

STRONG L^p ESTIMATES FOR MAXIMAL FUNCTIONS WITH RESPECT TO SINGULAR MEASURES; WITH APPLICATIONS TO EXCEPTIONAL SETS

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§1. Introduction. In this paper we prove some results on the boundary behavior of holomorphic and harmonic functions. At first sight, these results may seem unrelated. On the one hand, we are concerned with the boundary behavior of holomorphic functions in $H^p(D)$ along curves in the boundary of domains D in $\mathbf{C}^n$, and our object here is to generalize the earlier results in [9] and [10] for bounded holomorphic functions. On the other hand, we are interested in the existence of limits of Poisson integrals of L^p-potentials or functions in Besov spaces within tangential approach regions except on exceptional sets of appropriate Hausdorff measure zero. Our object here is to extend earlier results in [11] and [12], which dealt mainly with exceptional sets of Lebesgue measure zero.

We prove a sharp form of the quantitative results which implies the existence of these limits; that is, we prove the L^p boundedness, and not just weak-type, of the associated maximal functions. It is here that a common ingredient in the proofs of these results becomes apparent. This is the boundedness of certain (possibly tangential) maximal functions in L^p of singular measures in terms of the L^p norm of the standard nontangential maximal functions with respect to Lebesgue measure. This in turn depends on an atomic decomposition for certain tent spaces. This was proved in the Euclidean case by Coifman, Meyer, and Stein [2], and our use of this decomposition is motivated by the arguments in [12].

We need to apply this atomic decomposition in certain non-Euclidean nonisotropic situations however, and in the second section of this paper we show that there is such a decomposition in general spaces of "homogeneous type" as defined by Coifman and Weiss [3]. The proof is almost identical with the Euclidean case, but we include the proof for completeness. Then we use this to show that certain maximal operators are bounded in L^p of some (possibly singular) measures. This result is used in each of the remaining sections.

In the third section we show that if f is in H^p of a smoothly bounded domain in $\mathbf{C}^n$ then a certain maximal function involving the gradient over admissible approach regions is bounded in L^p of an appropriate singular measure. This allows us to get sharp gradient estimates for H^p functions near thin subsets of the

Received July 20, 1985. Supported in part by NSF grants at the University of Wisconsin, Madison.