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REGULARITY OF THE BERGMAN PROJECTION AND LOCAL GEOMETRY OF DOMAINS

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§1. Introduction. Let D be a bounded domain in $\mathbf{C}^n$ with smooth boundary. The Bergman projection for D is the orthogonal projection P from $L^2(D)$ onto the subspace consisting of all square-integrable holomorphic functions. This paper is concerned with global regularity properties of P; we would like to know, for example, under what circumstances P maps $C^\infty(\overline{D})$ into $C^\infty(\overline{D})$. Many important positive results in this line come from results on the $\bar{\partial}$-Neumann problem for domains whose boundaries satisfy some local geometric hypothesis involving the Levi form [12, 14, 7, 8]. (In particular pseudoconvexity is assumed.) On the other hand, there exist smooth bounded (nonpseudoconvex) domains in $\mathbf{C}^2$ for which P fails to map $C^\infty(\overline{D})$ into $L^{2+\epsilon}(D)$ for any positive ϵ [2]. It is reasonable, then, to ask if global regularity of P imposes any local geometric conditions on the boundary of D.

Earlier work of Bell shows that the geometry of interior boundary components of domains with globally regular Bergman projection can be entirely arbitrary [5]. (See also [16].) The results of this paper will show that the geometry of the outer boundary component is also unconstrained at the local level; more precisely, we shall establish the following theorem.

THEOREM 1. *If D is a domain in $\mathbf{C}^n$ with smooth boundary near a point $p \in bD$ then for every positive integer k there is a smooth bounded subdomain D_k of D such that*

(i) *The Bergman projection for D_k maps the Sobolev space $W^k(D_k)$ into itself (boundedly); and*

(ii) *There is a neighborhood U_k of p in $\mathbf{C}^n$ such that*

$$U_k \cap D = U_k \cap D_k .$$

Thus failure of regularity of P at any finite level of differentiability must stem from global considerations.

Theorem 1 is a consequence of global regularity estimates for the Bergman projection on domains which satisfy a hypothesis which is inconsequential at the local level while being quite stringent globally.

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