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CONTINUITY AND NODAL PROPERTIES NEAR INFINITY FOR SOLUTIONS OF 2-DIMENSIONAL SCHRÖDINGER EQUATIONS

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1. Introduction and statement of the results. In this paper we consider real valued $W^{2,2}$-solutions $\psi(x)$ of

$$(-\Delta + V - E)\psi = 0 \qquad \text{for} \quad x \in \Omega_R, \quad \Omega_R = \{x \in \mathbb{R}^2 : |x| = r > R\}, \quad R > 0 \tag{1.1}$$

Here the Sobolev space $W^{2,2}$ is defined as in [9]. Throughout the paper it will be assumed that $E < 0$ and that $V(x)$ fulfills the following conditions:

$$\begin{aligned} &\text{(A1)} && V(x) \text{ is real valued and continuous in } \overline{\Omega}_R \\ &\text{(A2)} && \lim_{r\to\infty} V(x) = 0 \\ &\text{(A3)} && \inf_{x\in\Omega_R}\left(V(x) - \frac{1}{4}|x|^{-2} - E\right) > 0. \end{aligned} \tag{A}$$

Clearly, (A1), (A2) and $E < 0$ imply that for some $\bar{R} \geqslant R$ $\inf_{x\in\Omega_{\bar{R}}}(V(x) - 1/4|x|^{-2} - E) > 0$. So without loss we shall assume (A3).

Considering $-\Delta + V - E$ on $C_0^\infty(\Omega_R)$ the above assumptions imply that $C_0^\infty(\Omega_R)$ is a core of the unique selfadjoint operator H_D associated to $-\Delta + V - E$ with Dirichlet boundary conditions on $\partial\Omega_R$ and due to (A3) H_D is positive definite. This guarantees that the Dirichlet problem in Ω_R is uniquely solvable given ψ on $|x| = R$. In particular it follows that $\int_{S^1}\psi^2(r,\omega)\,d\omega > 0$ for all $r \geqslant R$, with r, ω denoting polar coordinates (see e.g., [7] and also Theorem 1). Furthermore the above assumptions imply that ψ has continuous derivatives for $|x| > R$ (see e.g., [9]). We note that our assumptions on V are somewhat stronger then we shall need but will be kept for the sake of simplicity.

As in [11] we shall take a perturbational point of view and split V so that

$$V(x) = V_1(r) + V_2(x)$$

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