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THE GELFAND–NAIMARK THEOREM FOR JB^*-TRIPLES

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JB^*-triples (which will be defined later) occur in the study of bounded symmetric domains in finite and infinite dimensions. Kaup [21] showed the equivalence of the two categories: bounded symmetric domains in complex Banach spaces; and JB^*-triples. This result extended the work of Koecher [24], who introduced Jordan triple systems as a vehicle for classifying finite dimensional bounded symmetric domains. Koecher also showed their connection to symmetric Lie algebras, cf. [20].

A concrete example of a JB^*-triple, called a J^*-algebra, was introduced by Harris [15]. This class, which includes all C^*-algebras, all Jordan operator algebras, some Lie algebras and all Hilbert spaces, was shown to have the following property: the open unit ball is a homogeneous domain, i.e., the group of biholomorphic automorphisms acts transitively on it.

In [10] the authors showed that JB^*-triples occur naturally in the solution of the contractive projection problem for C^*-algebras. Since the morphism in this problem may not preserve order, its image takes us out of the category of C^*-algebras (whose morphisms are completely positive) and even out of the category of Jordan algebras (whose morphisms are positive) to a category based only on geometry and not depending on order structure, cf. [11]. Actually the above mentioned problem was solved by the authors in the wider category of J^*-algebras, and by Kaup [22] and Stachó [28] for JB^*-triples.

By combining the contractive projection theorem with the ultrafilter version of the principle of local reflexivity, Dineen [8] showed that the second dual of a JB^*-triple is itself a JB^*-triple. By refining the ultrafilter used in Dineen's proof, Barton and Timoney [5] showed that the extended triple product is separately weak*-continuous. Thus, the second dual of a JB^*-triple is a *JBW^*-triple*, i.e., a dual JB^*-triple with a separately weak*-continuous triple product.

The structures of a JBW^*-triple and its predual were studied by the authors in [12]. For example it was proved that each JBW^*-triple decomposes into a direct sum of atomic and purely nonatomic JBW^*-subtriples. Also, a study of JBW^*-triples was undertaken by Horn [18], who obtained a classification of type I JBW^*-triples. In particular Horn proved that the JBW^*-factors of type I are precisely the Cartan factors (these will be defined below).

Using these results, we are now able to prove the following Gelfand–Naimark type theorem.

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