Vol. 53, No. 1 DUKE MATHEMATICAL JOURNAL © March 1986

OSCILLATORY INTEGRALS AND SPHERICAL HARMONICS

CHRISTOPHER D. SOGGE

1. Introduction. In 1972 L. Carleson and P. Sjölin proved that if on $\mathbf{R}^2$ we define the Bochner–Riesz operators of index $\delta > 0$ by $(T^\delta f)\hat{}(\xi) = (1 - |\xi|^2)_+^\alpha \hat{f}(\xi)$ then one has

$$\|T^\delta f\|_{L^4(\mathbf{R}^2)} \leqslant C_\delta \|f\|_{L^4(\mathbf{R}^2)} . \tag{1.1}$$

One of our main goals is to extend the Carleson–Sjölin theorem to spherical harmonics on the two sphere, Σ^2. Namely, we show that if $f \in L^4(\Sigma^2)$ has the spherical harmonic development $f \sim \sum_{k=0}^\infty H_k f$ and if for $L \geqslant 0$ one defines

$$T_L^\delta f(\xi) = \sum_{0 \leqslant k \leqslant L} \left(1 - \left|\frac{k}{L}\right|^2\right)_+^\delta H_k f(\xi)$$

then for any fixed $\delta > 0$, if $L \geqslant 0$ one has the inequalities

$$\|T_L^\delta f\|_{L^4(\Sigma^2)} \leqslant C \|f\|_{L^4(\Sigma^2)} . \tag{1.1$'$}$$

One easily has inequality (1.1′) for any fixed $L \geqslant 0$, but the difficulty lies in showing the uniform bounds. Since it is a classical fact that a theorem for dilates of a zonal multiplier operator on Σ^n implies the corresponding radial multiplier on $\mathbf{R}^n$ (see e.g., [2]), we see that the family of inequalities (1.1′) actually implies (1.1). Also by using (1.1′) and analytic interpolation, we can establish a sharp theorem regarding Bochner–Riesz summation on $L^p(\Sigma^2)$, $1 \leqslant p \leqslant \infty$.

For technical reasons, we prove (1.1′) by establishing the corresponding result for the Cesàro summation operators (defined below), S_L^δ. This is done by first approximating the resulting kernels via some classical inequalities of Darboux–Szegö. Next one breaks these approximated kernels dyadically, applies a variant of the so-called "main lemma of Carleson–Sjölin," and adds up the resulting estimates.

In higher dimensions we are also able to prove sharp results for Cesàro summation on certain $L^p(\Sigma^n)$ spaces. To do this we use a result of Bonami–Clerc [2] which says that certain estimates for the (L^p, L^2) norm of the harmonic projection operator imply sharp results for Cesàro summation on $L^p(\Sigma^n)$. Their result (and argument) is analogous to that of Fefferman–Stein [14] which says

Received May 1, 1985. Revision received August 8, 1985. This work supported in part by a Sloan Doctoral Fellowship.