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PROOF OF THE ARNOLD CONJECTURE FOR SURFACES AND GENERALIZATIONS TO CERTAIN KÄHLER MANIFOLDS

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§1. Introduction and statement of the results. We consider a compact symplectic manifold (P, ω) with symplectic structure ω, which is a closed and nondegenerate 2-form. For example, every compact and oriented manifold of dimension 2 carries a symplectic structure given by any volume form ω. Since ω is nondegenerate, we can associate to a smooth function $H: S^1 \times P \to \mathbf{R}: (t, p) \to H_t(p)$ with $S^1 = \mathbf{R}/\mathbf{Z}$ a periodic family of vector fields X_t defined by

$$\omega = (\cdot, X_t) = dH_t .$$

This vector field is called the (exact) Hamiltonian vector field associated with the Hamiltonian H_t. Our aim is to find periodic solutions having period 1 of the differential equation

$$\frac{d}{dt} z(t) = X_t(z(t)). \tag{1.1}$$

We denote with $\phi: \mathbf{R} \times P \to P$, $(t, p) \to \phi^t(p)$ the flow of the vector field X_t; it is defined by the equations

$$\frac{d}{dt} \phi^t = X_t \circ \phi^t \quad \text{and} \quad \phi^0(x) = x.$$

Clearly the 1-periodic solutions of (1.1) are in one to one correspondence with the fixed points of the map $\psi = \phi^1$, which is a symplectic diffeomorphism. We conclude that there is at least one 1-periodic solution of (1.1) provided the Euler characteristic of P does not vanish. This follows from the Lefschetz fixed point theory applied to the map ψ, which is homotopic to the identity. However, the Lefschetz fixed point theory does not take into account the additional structure of ψ, which might guarantee additional fixed points.

Using different ideas it has been proved recently ([6]) that in fact on a torus T^{2n}, which has Euler characteristic zero, every exact Hamiltonian vector field possesses at least $2n+1$ periodic solutions with period 1, i.e., as many as a function on T^{2n} has critical points. This has been conjectured by V. Arnold in [18] and [19]. One might guess that, in general, every exact Hamiltonian vector

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