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ON THE RESTRICTION OF SUPERCUSPIDAL REPRESENTATIONS TO COMPACT, OPEN SUBGROUPS

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Let G be a reductive group over a p-adic field, F, let $Z(G)$ be the center of G, let K be a compact, open subgroup of G and let H be a subgroup of K which contains $Z(G)K$ with finite index. Then one is interested in determining the set of representations, η, of H for which the induced representation $\mathrm{Ind}_H^G\eta$ is supercuspidal (see, e.g., [Mau], [Shi], [Sha], [H_1], [K_1]). On the other hand, in attempting to show that *all* supercuspidal representations of G are of the form $\mathrm{Ind}_H^G\eta$ for appropriate choices of η and H it is helpful to know which representations τ of K have the property that no supercuspidal representation of G contain τ as a K-subrepresentation. The approach taken to characterizing such representations τ (called here *principal* representations) has generally been to show that any representation of G containing τ must have a nontrivial identity quotient upon restriction to the unipotent radical of some parabolic subgroup of G ([K_1], [Man]). A different approach is due to Matsumoto and was applied in the case of the metaplectic group by Blondel in [Bl].

Matsumoto's method applies when τ is a one-dimensional representation and uses the natural action of the Hecke algebra, $H(G,\bar{\tau})$, of $\bar{\tau}$-spherical functions on the τ-isotypical subspace, X_τ, of an admissible representation (π, X) of G to produce matrix coefficients for π which are not compactly supported modulo $Z(G)$. The purpose of this paper is to generalize Matsumoto's method to the case where τ is an arbitrary finite dimensional irreducible representation and then to give some applications.

We begin, in section one, with some general background. Our approach borrows heavily from [Bo] and [Cas] and details may be found in one of these sources. Our major observation here is that the natural action of $H(G,\bar{\tau})$ on X_τ should be understood by viewing π as a quotient of $\mathrm{Ind}_K^G\tau$, letting $H(G,\tau)$ act on the space of $\mathrm{Ind}_K^G\tau$ by right convolution and then identifying $H(G,\bar{\tau})$ with $H(G,\tau)$ via the natural antiisomorphism $f\mapsto\check{f}$ (see 1.7 below).

This suggests that the appropriate generalization is to work in the space, V, of $\mathrm{Ind}_K^G\tau$ and that, in particular, V_τ should be given the structure of a $(C_c^\infty(K), H(G,\tau))$ bimodule (see 1.5). Once having done this, we give our generalization of Matsumoto's result (Proposition 1.4) and several consequences.

In sections two and three, we apply the results of section one to show that any

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