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SL(2, C), H^1 AND SYMMETRIC TENSORS

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The purpose of this paper is to prove a theorem conjectured by Irwin Kra which is important for the study of relative Poincare series associated to loxodromic elements of Kleinian groups (see [1]). The following is needed to state the theorem. For every positive even integer d, let Π_d denote the $\mathbf{C}$-space of all polynomials over $\mathbf{C}$ of degree at most d. Make Π_d into a right PSL(2, C)-module so that if $\binom{r\ s}{t\ u} \in \mathrm{SL}(2,\mathbf{C})$ represents $g \in \mathrm{PSL}(2,\mathbf{C})$ and $P(z) \in \Pi_d$, then

$$P(z)g = P(g(z))(tz+u)^d.$$

THEOREM 0.1. *Let Γ be a nonelementary Kleinian group. Let χ be a cocycle for the Eilenberg–MacLane cohomology group $H^1(\Gamma, \Pi_d)$. Assume for every loxodromic* (eigenvalues having absolute value not 1) *$g \in \Gamma$ that there exists a polynomial $P \in \Pi_d$ such that*

$$\chi(g) = Pg - P.$$

Then χ is a coboundary.

This theorem follows from Theorem 1.3 which is somewhat more general.

I would like to thank Irwin Kra for introducing me to this problem and for many discussions concerning it. I would also like to thank Michio Kuga and Chih-Han Sah for numerous enlightening discussions (on numerous topics).

§1. The theorem and its proof. For what follows, it seems preferable to lift the group of Theorem 0.1 to SL(2, C) and to replace the (right) module by the (left) symmetric tensor module. So, let S denote the symmetric tensor algebra of $\mathbf{C}^2$. Let S_d denote its homogeneous component of degree d, and identify S_1 with $\mathbf{C}^2$.

It is also useful to have a nondegenerate bilinear form on S_d for what follows. The one used here appears in §8.2 of [3]. This form, denoted by $\langle\ ,\ \rangle$, is determined by the equation

$$\langle u^d, v^d \rangle = \det(u,v)^d \tag{1.1}$$

for every $u, v \in S_1 = \mathbf{C}^2$. This gives a nondegenerate bilinear form on S_d which is symmetric if d is even and antisymmetric if d is odd. Letting * denote adjoint with respect to this form and letting $\rho_d : \mathrm{GL}(2,\mathbf{C}) \to \mathrm{GL}(S_d)$ denote the symmetric

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