TRANSCENDENCE THEORY OVER FUNCTION FIELDS

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§1. Introduction. Let $\mathscr{C}$ be a smooth projective, geometrically irreducible curve over a finite field $\mathbf{F}_q$, $q = p^n$. We fix a rational point ∞ on $\mathscr{C}$, and consider the ring A of functions on $\mathscr{C}$ regular away from ∞. We set k to be the function field of $\mathscr{C}$ and k_∞ its completion at ∞. All elements of the algebraic closure $\bar{k}_\infty$ will be called "numbers". Elements in $\bar{k}_\infty$ which are transcendental over k are the transcendental numbers.

The transcendence theory here consists in determining the transcendency of values of certain special functions. Over global function fields, the most natural functions to consider are the functions introduced by Drinfeld [2]. He started from a lattice M, i.e., a discrete finitely generated A-module contained in $\bar{k}_\infty$, and formed the "exponential function":

$$e_M(z) = z \prod_{\substack{\omega \in M \\ \omega \neq 0}} \left(1 - \frac{z}{\omega}\right).$$

The functional equations satisfied by this function describe a Drinfeld module ϕ_M which is an $\mathbf{F}_q$-linear homomorphism from the Dedekind domain A into $\bar{k}_\infty\{F\}$, the noncommutative polynomial ring over $\bar{k}_\infty$ generated by $F: z \to z^q$ under composition.

If $K \subset \bar{k}_\infty$ is any subring and the image of ϕ_M is contained in $K\{F\}$, we say that the module ϕ_M is defined over K. For those Drinfeld modules which are defined over $\bar{k} \subset \bar{k}_\infty$, the coefficients of the Taylor expansion of $e_M(z)$ are also contained in $\bar{k}$. We have conjectured in [12] that if this is the case, then the corresponding entire function e_M should take transcendental values at all nonzero algebraic elements in $\bar{k}_\infty$. This conjecture is known true, [1], [8], only for the oldest Drinfeld modules—the so called carlitz modules—on the simplest curve $\mathscr{C} = \mathbf{P}^1(\mathbf{F}_q)$.

In this article we will make a step toward a general theory. We prove the following result for arbitrary curve $\mathscr{C}$ and for Drinfeld modules of arbitrary rank.

MAIN THEOREM. *Let ϕ_M be a Drinfeld module of rank d defined over $\bar{k}$ with corresponding lattice M. Let $\omega_1, \ldots, \omega_l$ be l k-linearly independent elements in $\bar{k}$ at which the function e_M assumes values also in $\bar{k}$. Then $l \leqslant d$.*

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