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CR EXTENSION NEAR A POINT OF HIGHER TYPE

A. BOGGESS AND J. PITTS

Section 1. Introduction, definitions and statement of the theorem. A long-standing problem in several complex variables is to examine the relationship between the convexity of a submanifold M of $\mathbf{C}^n$ and the local extendibility of CR functions on M to CR functions (or possibly even analytic functions) on a larger set. The geometric and analytic properties of this larger set are also of interest. The convexity of a submanifold is measured by its Leviforms. The classical Hans Lewy extension theorem [L] states that if the (first order) Leviform of a real hypersurface M at a point $p \in M$ is not identically zero, then CR functions on M near p extend to holomorphic functions on an open set $\tilde{\omega}$ in $\mathbf{C}^n$. If the eigenvalues of the Leviform have the same sign, then $\tilde{\omega}$ lies to one side of M, and if there are two eigenvalues of opposite sign, then $\tilde{\omega}$ lies on both sides of M (i.e., $\tilde{\omega}$ contains p). Hill and Taiani [HT, theorems 9.1 and 10.1] generalize this result to the case when $\operatorname{codim}_{\mathbf{R}} M > 1$. In this case, they show $\tilde{\omega}$ is a manifold with boundary with $\dim_{\mathbf{R}} \tilde{\omega} = \dim_{\mathbf{R}} M + 1$, and they show that CR functions on M extend to CR functions on $\tilde{\omega}$. Their manifold $\tilde{\omega}$ is roughly one third as smooth as M. Now, in general $\tilde{\omega}$ is not unique. In this paper, we construct an $\tilde{\omega}$ with only an ϵ-loss of derivatives (a precise statement of the smoothness of $\tilde{\omega}$ is given in the theorem below). Moreover, Hill and Taiani only consider the first order Leviform. There exist higher order Leviforms which measure higher order convexity, and we only require the nonvanishing of one of the Leviforms of order ℓ for some $\ell \geqslant 1$. These results were announced in [BP].

The construction of $\tilde{\omega}$ requires a careful analysis of the generalized Bishop's equation which we think will prove valuable in other CR extension problems. It is our pleasure to thank J. C. Polking who among other things helped formulate the generalized Bishop's equation.

Now we shall define notation.

Suppose M is a C^1 submanifold of $\mathbf{C}^n$ of real codimension d. We let $T(M)$ be the real tangent bundle to M with fiber $T_p(M)$, $p \in M$. We let $H_p(M)$ be the holomorphic tangent space of M at p. We assume throughout that M is *CR* and *generic* which means that $\dim_{\mathbf{R}}\{H_p(M)\}$ is minimal (i.e., $2n - 2d$) for all $p \in M$. For $p \in M$, we let $Y_p(M) = T_p(M)/H_p(M)$. Using the standard metric for $\mathbf{R}^{2n} = \mathbf{C}^n$ we have $T_p(M) = H_p(M) \oplus Y_p(M)$, where this is an orthogonal direct sum.

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