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PLURI-CANONICAL DIVISORS ON KÄHLER MANIFOLDS II

MARC LEVINE

Introduction. In our paper [L] we proved the following theorem:

THEOREM. *Let $p: X \to D$ be a smooth and proper map of a complex manifold X to a disk D, with connected fibers. Fix a positive integer m. Suppose that the fiber $X_0 = p^{-1}(0)$ is in the class $\mathscr{C}$ of Fujiki. Suppose further that the general element s of $H^0(X_0, \mathcal{O}_{X_0}(mK))$ has smooth divisor. Then the m-genus $P_m(p^{-1}(t))$ is constant over a neighborhood of 0 in D.*

Here we will prove an extension of the above theorem to the case in which the general m-canonical divisor has singularities. More precisely, let s be a general element of $H^0(X_0, \mathcal{O}_{X_0}(mK))$, let Y be the m-fold covering of X_0, contained in the canonical line bundle, and branched over (s). Let $f: Y^* \to Y$ be a resolution of singularities of Y such that the exceptional locus E is a divisor with normal crossing. Let ω be the dualizing sheaf on Y. Then Y has "mild" singularities if

(1) Y is smooth in codimension one (i.e., (s) is reduced)

(2) $f^*(\omega)$ is contained in the sheaf of forms with log poles $\Omega^r_{Y^*}\langle E\rangle$, $r = \dim(Y)$

For example, if (s) is a reduced divisor with normal crossing, then Y has "mild" singularities.

We prove here the following theorem:

THEOREM. *Let $p: X \to D$ be a smooth and proper map of a complex manifold X to a disk D, with connected fibers. Fix a positive integer m. Suppose that X_0 is in $\mathscr{C}$, and suppose further that for a general element s of $H^0(X_0, \mathcal{O}_{X_0}(mK))$, the m-fold covering Y of X_0 branched along (s) has "mild" singularities. Then the m-genus $P_m(p^{-1}(t))$ is constant over a neighborhood of 0 in D.*

Let $p: Z \to D$ be a smooth map of a complex manifold Z to a disk D, and let t be a parameter on D. We denote by Z_n the reduction of Z mod t^{n+1},

$$\mathcal{O}_{Z_n} = \mathcal{O}_Z/(t^{n+1}).$$

We define the sheaf of relative C^∞ functions on Z_n, $\mathscr{C}_{Z_n}^{(0,0)}$, by

$$\mathscr{C}_{Z_n}^{(0,0)} = \mathscr{C}_Z^\infty/(t^{n+1}, \bar{t}).$$

If U is a coordinate patch on Z, with coordinates (z,t), $z = (z_1, \dots, z_d)$, and x is

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