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SYMMETRY OF CONSTANT MEAN CURVATURE HYPERSURFACES IN HYPERBOLIC SPACE

GILBERT LEVITT AND HAROLD ROSENBERG

Introduction. In a recent paper, M. Do Carmo and B. Lawson studied hypersurfaces M of constant mean curvature in hyperbolic space [2]. They use the Alexandrov reflection technique to study M given the asymptotic boundary $\partial_\infty M$. For example, one of their theorems says M is a horosphere when $\partial_\infty M$ reduces to a point. They also prove a Bernstein type theorem for minimal graphs.

In this paper we shall extend their results to other boundary conditions. We prove an embedded M, of constant mean curvature, with $\partial_\infty M$ a subset of a codimension one sphere S, either is invariant by reflection in the hyperbolic hyperplane H spanned by S or is a hypersphere. In the former case M is a "bigraph" over H: it meets any geodesic orthogonal to H either not at all or transversely in two points (one on each side of H) or tangentially on H.

As a corollary of this, when $\partial_\infty(M)$ consists of two points p and q, then M is a hypersurface of revolution about the geodesic joining p to q.

We also consider minimal immersed hypersurfaces $M \subset H^n$ with M regular at ∞. When $\partial_\infty M$ consists of two disjoint spheres S_1, S_2 we prove M is a catenoid or M is the union of the two hyperbolic planes spanned by S_1 and S_2.

The principal techniques we use to obtain these results are the Alexandrov reflection principle and R. Schoen's adaptation of this to complete minimal surfaces [4].

I. Definitions and notations. When we refer to plane, distance, line, etc. we always mean the hyperbolic object in H^n. We work with the *Poincaré model* of H^n: H^n is the interior of the unit ball in R^n. The asymptotic boundary of H^n is identified with the boundary of the unit ball and denoted by $S(\infty)$. Given $A \subset H^n$, we denote by $\partial_\infty A$ the set of accumulation points of A in $S(\infty)$ and call it the *asymptotic boundary* of A. When the context is clear, we will omit the subscript ∞.

Fix a hyperplane P_0 in H^n. We have two natural coordinate systems. First, one can use the geodesics orthogonal to P_0 to give each point coordinates (x, t) where $x \in P_0$ and t is the distance from x to (x, t). This system does not suit our purposes because translation along one geodesic orthogonal to P_0 does not leave invariant another such geodesic. Also this does not extend to a coordinate system on $S(\infty)$.

Instead we shall use the *latitude-longitude system*. More precisely, choose

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