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THE ASYMPTOTIC EXPANSION OF MINAKSHISUNDARAM–PLEIJEL IN THE EQUIVARIANT CASE

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1. Introduction. In 1912 H. Weyl [24] determined the asymptotic behavior of the eigenvalues of the Laplacian for a compact domain in $\mathbb{R}^3$. Almost forty years later Minakshisundaram and Pleijel generalized this to a full asymptotic expansion for the corresponding Dirichlet series for a compact Riemannian manifold [20], giving rise to an extremely fruitful new development based on the identification of the coefficients in the expansion (see e.g. [16], [18], [4], [12], [2]). One should expect interesting information from an extension of these methods to singular spaces. In this paper we extend the Minakshisundaram–Pleijel expansion to the equivariant case. Even in the simplest nontrivial cases the structure of the coefficients becomes very complicated so we concentrate here on the existence of an expansion and the functions involved.

To describe the results let M be a compact n-dimensional Riemannian manifold, G a compact group of isometries, and ρ a finite dimensional irreducible representation of G in a complex vector space V. Denote by E_λ the complexified eigenspace of the negative Laplacian $-\Delta$ with eigenvalue λ, $\lambda \geqslant 0$. Then E_λ is G-invariant and we can consider the function

$$L_\rho(t) := \sum_{\lambda \geqslant 0} e^{-\lambda t} \dim \operatorname{Hom}_G(V, E_\lambda), \qquad t > 0.$$

Note that $\dim \operatorname{Hom}_G(V, E_\lambda)$ is the multiplicity of ρ in E_λ which is equal to $\dim E_\lambda^G$ in case ρ is the trivial representation. Our main result (Theorem 4 below) states that L_ρ has an asymptotic expansion as $t \to 0$ of the form

$$L_\rho(t) \sim (4\pi t)^{-m/2} \sum_{\substack{j \geqslant 0 \\ 0 \leqslant k \leqslant K_0 - 1}} a_{jk} t^{j/2} (\log t)^k$$

where $m := \dim {}^M/_G$ and K_0 is bounded by the number of different dimensions of G-orbits in M. Recall that the union of principal orbits, M_0, is open and dense in M and that ${}^{M_0}/_G$ is a manifold whose dimension is $\dim {}^M/_G$ by definition. If G is trivial Theorem 4 gives the classical result of Minakshisundaram and Pleijel mentioned above. If G has no singular orbits (and hence $K_0 = 1$) it is contained

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