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STABLE TRACE FORMULA: CUSPIDAL TEMPERED TERMS

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Consider a connected reductive group G over a number field F. For technical reasons we assume that the derived group of G is simply connected (see [L1]). In [L3] Langlands partially stabilizes the trace formula for G. After making certain assumptions, he writes the elliptic regular part of the trace formula for G as a linear combination of the elliptic G-regular parts of the stable trace formulas for the elliptic endoscopic groups H of G. The function f^H used in the stable trace formula for H is obtained from the function f used in the trace formula for G by transferring orbital integrals.

Langlands uses $\iota(G,H)$ to denote the coefficient of the stable trace formula of H in the linear combination referred to above. He obtains an explicit formula for $\iota(G,H)$, which we review in Section 8.

Eventually it should be possible to stabilize the whole trace formula for G. In Section 12 we sketch the stabilization of the cuspidal tempered part of the trace formula for G. To carry this out we are forced to make a number of assumptions, some of which are unlikely to be verified in the near future, but this is not as serious as it appears at first, for our only purpose in Section 12 is to increase our understanding of the formalism that underlies the stable trace formula. In any case, we are able to prove a small part of what we need, and we put these results in Section 11.

Although the stabilization carried out in Section 12 is speculative, it leads us to a new expression for $\iota(G,H)$. In Section 8 we prove that this expression agrees with the one found by Langlands. This agreement is encouraging and should, perhaps, be regarded as evidence in favor of the assumptions in Section 12. Our expression for $\iota(G,H)$ is

$$\iota(G,H) = \tau_1(G)\cdot\tau_1(H)^{-1}\cdot\lambda^{-1}.$$

The number λ also appears in Langlands's expression for $\iota(G,H)$; its definition can be found in 8.1. The number $\tau_1(G)$ is given by

$$|\pi_0\big(Z(\hat{G})^{\Gamma}\big)|\cdot|\ker^1\big(F,Z(\hat{G})\big)|^{-1}.$$

We need to explain the symbols used in this formula. We write $\hat{G}$ instead of ${}^LG^0$,

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