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ON THE K-THEORY OF SURFACES WITH MULTIPLE CURVES AND A CONJECTURE OF BLOCH

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Introduction. The purpose of this paper is to discuss the extension to certain singular complex surfaces of the following conjecture of Bloch ([3]):

CONJECTURE. *Let X be a nonsingular projective surface over a field k of characteristic zero. Then if $p_g = \dim H^2(X, \mathcal{O}_X) = 0$ the Chow group* $\mathrm{CH}^2(X)_0$ *of 0-cycles of degree zero modulo rational equivalence is finite dimensional.*

Without discussing in detail what is meant here by 'finite dimensional' we can say that it is known that all reasonable definitions are equivalent to asserting that the natural map

$$CH^2(X)_0 \to k\text{-pts of } \mathrm{Alb}(X)$$

is an isomorphism, or at least an isogeny (i.e., surjective with finite kernel) ([5], [28]). Bloch's conjecture is known to be true for all X of special type, and some X of general type ([4], [5]). It should be pointed out that the original result in this area is due to Mumford ([26]), and says that if $p_g \neq 0$, then $CH^2(X)_0$ cannot be finite dimensional.

Bloch's conjecture was based on the interpretation of the Chow group via K-theory:

$$CH^2(X) \simeq H^2(X, K_2(\mathcal{O}_X)).$$

(see [3], [21] for more details on the connection between this formula and the conjecture); and since the groups $H^*(X, K_*(\mathcal{O}_X))$ seem to provide a good intersection theory for singular varieties (cf. [9]), it seems reasonable to discuss whether Bloch's conjecture is still true when X is singular. We shall consider the

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