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EQUIVARIANT K-THEORY FOR CURVES

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Let X be a smooth curve defined over an algebraically closed field k and let $G \subseteq \mathrm{Aut}_k(X)$ be a finite reductive subgroup, that is, the order of G is not divisible by char(k).

We considered the equivariant K-group $K_G^{\cdot}(X)$ in [6] in order to give a natural proof of a Lefschetz–Riemann–Roch formula for G-linearised sheaves on X, that generalises Chevalley–Weil's formula for the representation of G on the holomorphic m-differentials, [3], [12]. They were inspired by previous computations of Hecke on the congruence modular curves [7].

After a request for a concrete computation of $K_G^{\cdot}(X)$, the second author gave some approximate formulas in [8], in the case where X is projective, sufficient for the applications in mind.

Here, we continue the computations of [8] and give a precise description of $K_G^{\cdot}(X)$, without the projectivity assumption on X. Set $Y = X/G$. Then we first describe the additive structure in terms of $K^{\cdot}(Y)$ and the representation rings of the ramification groups (Thm. 1.7).

Departing from this we give a description of the $R_k(G)$-module structure of $K_G^{\cdot}(X)$ (Thm. 2.2), and finally we determine the ring structure of $K_G^{\cdot}(X)$, giving generators and relations for it as a $K^{\cdot}(Y)$-algebra (Thm. 3.1).

As an example we work out the $R_k(G)$-module structure, when X is a projective hyperelliptic curve, and G is the group of order 2 generated by the canonical involution.

Our original interest in the subject stems from the applications to the structure of the moduli space for algebraic curves. Here the full representation on $H^0(X, \Omega^{\otimes 2})$ comes in, or rather, $H^1(X, \theta)$.

Of course, many others have considered equivariant K-theory, [1], [11], [10], [9], [2]. However, except for [2], with most success in the commutative case, these papers were almost exclusively concerned with a Lefschetz–Riemann–Roch formula with values in a topological (co-)homology theory. This was not sufficient for our application.

The first author has undertaken the study of a variation of the Lefschetz-trace in the nonreductive case in [4] and [5], that generalises previous work in that direction. We hope to come back to a full elaboration of this, also in a number-theoretical context.

0. Notations. Let k, X, G and $Y = X/G$ be as above, and denote the canonical morphism $X \to Y$ by π. The branch locus of π is denoted by B. Recall

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