Vol. 50, No. 4 DUKE MATHEMATICAL JOURNAL © December 1983

THETA FUNCTIONS AND HOLOMORPHIC DIFFERENTIAL FORMS ON COMPACT QUOTIENTS OF BOUNDED SYMMETRIC DOMAINS

GREG W. ANDERSON

Introduction. For positive integers p and q put

$$X_{p,q} \stackrel{\text{def}}{=} \{ z \in M_{p,q}(\mathbf{C}) \mid 1_q > {}^t\bar{z}z \} \simeq SU(p,q)/S(U(p) \times U(q)),$$

$$X_q^+ \stackrel{\text{def}}{=} \{ z \in X_{q,q} \mid {}^tz = z \} \simeq Sp(q,\mathbf{R})/U(q),$$

$$X_q^- \stackrel{\text{def}}{=} \{ z \in X_{q,q} \mid {}^tz = -z \} \simeq SO^*(2q)/U(q).$$

Our principal results are the following:

THEOREM A. *Let p, q, r and s be integers such that $p,q > 0$, $0 \leqslant r \leqslant p$, $0 \leqslant s \leqslant q$. Then there exists a compact Kähler manifold V uniformized by $X_{p,q}$ for which*

$$H^{(pq-rs,0)}(V) \neq \{0\}.$$

THEOREM B. *Let q and r be integers such that $q > 0$, $q \geqslant r \geqslant 0$. Then there exists a compact Kähler manifold V uniformized by X_q^+ for which*

$$H^{((1/2)(q+1)q-(1/2)(r+1)r,0)}(V) \neq \{0\}.$$

THEOREM C. *Let q and r be integers such that $q > 1$, $q \geqslant r \geqslant 1$. Then there exists a compact Kähler manifold V uniformized by X_q^- such that*

$$H^{((1/2)(q-1)q-(1/2)(r-1)r,0)}(V) \neq \{0\}.$$

The vanishing theorems of Parthasarathy [P] forbid the existence of nonzero holomorphic differential forms on compact quotients of the bounded symmetric domains $X_{q,q}$, X_q^+, and X_q^- in all the degrees other than those for which we can produce examples; in that sense our result is sharp.

The construction of nonvanishing holomorphic differential forms on compact quotients of bounded symmetric domains has been studied by a number of

Received December 11, 1982. The material in this paper is derived from the author's 1980 Princeton doctoral dissertation.