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REMARKS ON OUR PAPER "A SCATTERING THEORY FOR TIME-DEPENDENT LONG-RANGE POTENTIALS"

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§1. Introduction. In our recent paper [4], which will be referred to as [I] hereafter, we studied the scattering theory for the Schrödinger equations with time-dependent long-range potentials

$$i\partial u/\partial t = H(t)u \equiv -(1/2)\Delta u + V(t)u. \tag{1.1}$$

We proved the existence of the modified wave operators and then characterized their image in terms of the space-time behavior of the wave packet. Particularly, when $V(t)$ is periodic in time, we identified them with the continuous subspace of the Floquet operator associated with (1.1). The purpose of this paper is to present a simpler proof of a stronger version of the main theorem of [I] exploiting a compactness argument which is reminiscent of the Mourre approach [5] to the Enss method [1] (cf. also [2] and [6]).

We shall assume exactly the same assumption as in [I]. For the readers' convenience we repeat it here: We use the same notation as in [I].

Assumption (L). (i) $V^L(t,x)$ is a real-valued function of $(t,x) \in R^{N+1}$.

(ii) For each $t \in R^1$, $V^L(t,\cdot) \in C^\infty(R^N)$.

(iii) For any multi-index α, $\partial_x^\alpha V^L(t,x) \in C(R^{N+1})$.

(iv) There exists a constant $0 < \epsilon < 1$ such that for any α,

$$|\partial_x^\alpha V^L(t,x)| \leqq C_\alpha \langle x\rangle^{-|\alpha|-\epsilon}, \qquad (t,x) \in R^{N+1}.$$

Assumption (S). (i) For any $t \in R^1$, $V^s(t)$ is a symmetric operator in $\mathscr{H} = L^2(R^N)$ with domain $\mathscr{D}(V^s(t)) \supset H^2(R^N)$.

(ii) For some $0 < \sigma < 1$ and $0 \leqq \delta < 1/2$,

$$\sup_{t \in R^1} \| V^s(t)\langle x\rangle^{\sigma+1}(-\Delta+1)^{-\delta}\| < \infty.$$

(iii) For $f \in \mathscr{S}$, $V^s(t)f$ is an $\mathscr{H}$-valued continuous function.

Assumption (G). There exists a family of unitary operators $\{U(t,s)\}_{(t,s)\in R^2}$ in $\mathscr{H}$ satisfying the following properties.

(i) $U(t,s)$ is a strongly continuous function of $(t,s) \in R^2$.

(ii) $U(t,r)U(r,s) = U(t,s)$ for all $t,r,s \in R^1$.

(iii) There exists a subspace $\mathscr{Y} \subset \mathscr{H}$ with $\mathscr{S} \subset \mathscr{Y} \subset H^2(R^N)$ such that

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