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INDUCED REPRESENTATIONS AND THE COHOMOLOGY OF DISCRETE SUBGROUPS

BIRGIT SPEH

Introduction. Around 1971 G. Harder started to consider nontrivial cohomology classes of a discrete arithmetic torsion-free rank one subgroup Γ of a semisimple Lie group G which can be represented by Eisenstein series [4][5]. These classes have the following property.

Let P be a parabolic subgroup of G with Langlands decomposition $P = MAN$. Suppose we have an embedding

$$E : \operatorname{ind}_P^G \pi \otimes \nu \to C^\infty(G/\Gamma)$$

constructed via non singular Eisenstein series. Here π is a representation of M and ν a character of AN. This map induces a map on cohomology

$$E^* : H^*(\mathfrak{g}, K, \operatorname{ind}_P^G \pi \otimes \nu) \to H^*(\mathfrak{g}, K, C^\infty(G/\Gamma))$$

where K is a maximal compact subgroup of G and $\mathfrak{g}$ is the Lie algebra of G. Then the cohomology classes obtained by G. Harder are in the image of E and have a non-trivial restriction to the cohomology of the boundary of the Borel–Serre compactification of G/Γ [1].

In this paper we study the map E^* and in particular its kernel for arbitrary semisimple Lie groups and discrete torsion-free finitely generated subgroups Γ more closely.

Suppose $P = LN$ is a rational parabolic subgroup and Γ_L the projection of $\Gamma \cap P$ under the map $P \to L \cong P/N$. Suppose π is an irreducible cuspidal representation of L/Γ_L and χ a character of L with values in $\mathbf{C}$. Considering $\pi \otimes \chi$ as a representation of P we define

$$I(P, \pi, \chi) = \operatorname{ind}_P^G \pi \otimes \chi,$$

If $H^*(\mathfrak{g}, K, I(P, \pi, \chi)) \neq 0$, then

$$H^*(\mathfrak{g}, K, I(P, \pi, \chi)) \cong H^*(\mathfrak{l}, K \cap P, \pi) \otimes \Lambda^* \mathfrak{a}$$

where $\mathfrak{a}$ is a maximal abelian semisimple subalgebra of $\mathfrak{l}$.

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