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A NEW PROOF OF THE MOURRE ESTIMATE

RICHARD FROESE AND IRA HERBST

1. Introduction. Suppose H is an N-body quantum Hamiltonian and $E(\Delta)$ is the spectral projection for H onto the interval Δ. Let A denote the generator of dilations. Then the Mourre estimate is the statement

$$E(\Delta)[H,A]E(\Delta) \geqslant \alpha E(\Delta) + K \tag{1.1}$$

where Δ is a sufficiently small interval about a nonthreshold point, α is positive and K is compact. It was Mourre [5] who first proposed this estimate, proved it for 3-body Hamiltonians, and showed how to use it as a first step for proving absence of singular continuous spectrum. For this reason we call it "the Mourre estimate". However, the first proof for a large class of N-body Hamiltonians for arbitrary N was given by Perry, Sigal, and Simon in their paper on spectral analysis of N-body Schrödinger operators [6]. These authors then used the Mourre estimate to prove absence of singular continuous spectrum [6]. Their paper also contains additional historical remarks. Recently the present authors used (1.1) to prove a variety of results including the nonexistence of positive eigenvalues [3].

We will consider the class of Schrödinger operators of the form $-\Delta + \sum_i V_i(\pi_i(x))$ where π_i are projections onto subspaces on which the V_i are defined and go to zero in some sense. This is a natural class of operators to which to apply the geometrical ideas [2, 7, 8, 11] which have become prominent in the analysis of N-body quantum Hamiltonians. These operators have been considered before, for example by Agmon [1], who proved detailed exponential upper bounds for eigenfunctions of such operators, and in [6] Perry, Sigal, and Simon state that their method, with a suitable definition of thresholds, extends to handle them. We find that a subspace semilattice generated by the $\mathrm{Ran}(\pi_i)$ provides a natural partial order which can be used to prove the Mourre estimate by induction. Thus we avoid the complicated cluster expansions occurring in [6].

At first glance our assumptions on the potentials seem slightly weaker than those given by Perry, Sigal, and Simon in [6]. However, these authors chose to make one set of assumptions on the potentials under which all the results in their paper could be proved: their proof of the Mourre estimate actually goes through under our assumptions.

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