Vol. 49, No. 4 DUKE MATHEMATICAL JOURNAL © December 1982

ASYMPTOTIC BEHAVIOR OF MATRIX COEFFICIENTS OF ADMISSIBLE REPRESENTATIONS

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Introduction. In this paper we will restate and reprove several old, but largely unpublished results of Harish-Chandra ([11], [12], [13], [22]) regarding the behavior at infinity of matrix coefficients of certain representations of reductive Lie groups. Our methods are rather different from those of Harish-Chandra. Very briefly put, the difference is that we make a coordinate change that allows us to formulate things in terms of systems of complex differential equations and thus apply elegant but elementary results of Deligne [8].

More precisely, let G be a reductive group in what we call the Harish-Chandra class (see Section 1), K a maximal compact subgroup and $\mathfrak{g}$ the complexified Lie algebra of G. Suppose that (π, V) is a smooth representation of G annihilated by an ideal I of finite codimension in $\mathfrak{Z}(\mathfrak{g})$, the center of the enveloping algebra $\mathfrak{U}(\mathfrak{g})$ of $\mathfrak{g}$. We will be concerned with a description of the matrix coefficient $\langle \pi(x)v, \tilde{v}\rangle$ as $x \in G$ tends to infinity, where v is a K-finite vector in V and $\tilde{v}$ a K-finite vector in the dual $\tilde{V}$ of V. If θ is a Cartan involution of G associated to K and A a maximal θ-stable closed vector subgroup of G, then $G = KAK$; so that, because of the K-finiteness assumption one may as well assume $x \in A$. Loosely put, the K-finiteness of v and $\tilde{v}$, together with the assumption that I

Received June 30, 1982. Casselman partially supported by the National Science and Engineering Research Council of Canada and Miličić partially supported by the National Science Foundation and Sonderforschungsbereich "Theoretische Mathematik", Universität Bonn, FRG.