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ON AN EXAMPLE OF MANIN

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1. Introduction. Let V be a cubic surface defined over a field K and $V(K)$ the set of nonsingular points of V, where by "points" we mean geometric points with values in K.

We recall a few definitions and results of Manin [1].

1.1 Three points $x_1, x_2, x_3 \in V(K)$ are said to be *colinear* if they lie on some straight line L defined over K and either $L \subset V$ or $L \cdot V = x_1 + x_2 + x_3$ is a cycle of intersection of V with L.

Points $x_1, x_2 \in V(K)$ are said to be *in general position* if $x_1 \neq x_2$ the straight line x_1x_2 is not tangent to V and does not lie on V.

A point $x \in V(K)$ is an *Eckhardt point* if, considered over the algebraic closure of K, the tangent plane to V at x intersects V in a curve which decomposes into three straight lines all passing through x.

A point $x \in V(K)$ is a point *of general type* if there is no straight line on V that passes through x.

An equivalence relation A on $S \subset V(K)$ is said to be *admissible* if the following conditions hold:

(i) $\forall x_1, x_2 \in S$, $\exists x_3 \in S : x_1, x_2, x_3$ are collinear.

(ii) If x_1, x_2, x_3 and $y_1, y_2, y_3 \in S$ are collinear, $x_1 \sim y_1 \bmod A$ and $x_2 \sim y_2 \bmod A$, then $x_3 \sim y_3 \bmod A$.

The finest equivalence relation on S satisfying (i) and (ii) is called *universal equivalence*.

Let A be an admissible equivalence relation on S and let $E = S/A$ denote the set of its equivalence classes. For any $X_1, X_2 \in E$ there is a uniquely determined class $X_3 = X_1 \circ X_2 \in E$ such that there exist collinear points x_1, x_2, x_3 with $x_i \in X_i$. Defining a new binary operation $XY = U \circ (X \circ Y)$ where U is a fixed class, one converts E into a commutative Moufang loop (for brevity, we shall speak of CML E). If $S = V(K)$, then CML E is the direct product of an abelian group of period 2 and a CML of period 3 (the latter is also an abelian group if it is of order at most 27).

1.2 Now let K be either a finite extension of a p-adic number Q_p or a field of formal power series with a finite field of residues; let O be the ring of integers of

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