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ON GENERALIZATION OF THEOREMS OF A. D. ALEXANDROV AND C. DELAUNAY ON HYPERSURFACES OF CONSTANT MEAN CURVATURE

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Introduction. Among those local differential geometric invariants of a given submanifold of a Riemannian space, the mean curvature is undoubtedly the simplest basic invariant with obvious geometric meaning. Due to the natural physical interpretation as "soap bubbles" as well as their geometric simplicity and elegance, hypersurfaces of constant mean curvature in the euclidean, spherical or hyperbolic spaces have always been a natural, basic topic of investigation in the study of differential geometry of submanifolds. In 1841 [3], C. Delaunay discovered a remarkable way of constructing the generating curves of those rotational surfaces of constant mean curvature in the euclidean 3-space E^3, namely, tracing the locus of a focus by rolling a given conic section on a line (the rotational axis). In a recent work of the author and W. Yu [7], we generalized the above Delaunay's rolling construction to the case of rotational hypersurfaces of constant mean curvature in euclidean n-space E^n. As an immediate consequence of such construction, all rotational hypersurfaces of constant mean curvature in E^n, $n \geqslant 3$, except the special cases of spheres or minimal hypersurfaces, are necessarily *periodic*.

The characterization of spheres as the only imbedded closed hypersurfaces of constant mean curvature in E^n has a long and interesting history. The first result in this direction was obtained by Liebmann in 1900 [8] for the case of E^3 and under the restrictive added assumption of strict convexity. In 1958 [1], A. D. Alexandrov finally succeeded in proving that the equi-distance spheres are the only possible shape of imbedded, closed, hypersurfaces of constant mean curvature in the euclidean or hyperbolic n-space. The proof of Alexandrov uses an ingenious geometric argument exploiting the rich reflectional symmetries of constant curvature spaces. In essence, his proof amounts to show that a bounded, imbedded, complete, hypersurface of constant mean curvature in an euclidean or hyperbolic space already possesses enough reflectional symmetries to generate a subgroup of $\mathcal{O}(n)$-type and hence must be an $\mathcal{O}(n)$-orbit, i.e., an equi-distance sphere with respect to the fixed point of $\mathcal{O}(n)$. Notice that in the construction of C. Delaunay, the hypersurfaces of constant mean curvature are *assumed* to be

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