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TWO ALGEBRAS OF BOUNDED FUNCTIONS

THOMAS H. WOLFF

We will study two function algebras QC and QA which arise when one looks at function theory on the unit disc in $\mathbf{C}$ from the point of view of functional analysis. Denote bounded measurable functions on the circle $\{z \in \mathbf{C} \mid |z| = 1\}$ by L^∞, boundary values of bounded analytic functions on the disc by H^∞, and continuous functions by C; then Sarason [12] showed that the linear span of H^∞ and C is a uniformly closed subalgebra of L^∞. Moreover, $H^\infty + C$ is the smallest such algebra which properly contains H^∞. Together with $H^\infty + C$ one considers QC, the largest C^* algebra contained in $H^\infty + C$, and $QA = QC \cap H^\infty$. In [14], Sarason showed that QC is the intersection of L^∞ with the class VMO of functions whose mean oscillations over arcs tend to zero with the length of the arc.

Most results of this paper are analogues for QA of known theorems about the disc algebra A. A reference for these is chapter 6 of [9]. Our main lemma is the following.

THEOREM 1. *Suppose $f \in L^\infty$. There is an outer function $q \in QA$ such that $qf \in QC$.*

This should be compared with Fatou's theorem that any closed set of measure zero on the circle is the zero set of a function in the disc algebra. Another formulation of Fatou's theorem would be that theorem 1 remains true with QA and QC replaced by A and C, provided that f is continuous off a closed set of measure zero. We prove theorem 1 in section 1 and in section 2 we use it to prove some other results about QA, by modifying the arguments that lead from Fatou's theorem to the other standard theorems about the disc algebra. We then observe that our results can be combined with those of Stegenga [16] to answer a question of Sarason [13] about $H^\infty + C$. This is theorem 2.

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1. Proof of Theorem 1.

Notation D is the disc $\{z \in \mathbf{C} \mid |z| < 1\}$ and ∂D its boundary. If $E \subset \partial D$, then $|E| = \int_E d\theta$ is the Lebesgue measure of E, normalized so that $|\partial D| = 2\pi$. If

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