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COLLARS IN KLEINIAN GROUPS

ROBERT BROOKS AND J. PETER MATELSKI

Let $M = \mathsf{H}^3/G$ be the quotient space of hyperbolic 3-space by a discrete group G of isometries of hyperbolic space. M is then a complete hyperbolic 3-manifold, with singularities if G contains torsion elements. If γ is a simple closed geodesic in M, then there is a tubular neighborhood about γ, i.e., for some $r(\gamma) > 0$ the exponential map of the normal bundle $N(\gamma)$ into M is injective for all $x \in N(\gamma)$, $|x| < r(\gamma)$.

The purpose of this paper is to give a lower bound for the size of $r(\gamma)$, depending only on the length of γ and the "twist" about γ, and independently of M.

The fact that such a bound exists independent of M is a consequence of an inequality of Jørgensen [4] concerning the discreteness of 2-generator subgroups of $PSL(2, \mathsf{C})$, the group of isometries of hyperbolic space, as we will explain in §1 below. Indeed, we will show in §2 how Jørgensen's inequality leads to an explicit lower bound for $r(\gamma)$ when γ is sufficiently short and the "twist" about γ is small. It will follow from our results that $r(\gamma)$ tends to ∞ as the length of γ tends to 0, independent of the "twist" about γ. This has also been studied by T. Jørgensen, A. Marden and R. Meyerhoff. We prove a number of further results pertaining to the geometry of these tubular neighborhoods in M and to the sharpness of the estimates.

In particular, it is shown that as a family of loxodromic motions tend to a parabolic, the corresponding tubular neighborhoods tend to the standard horocycle of the parabolic, as we explain in §2 and §3 below.

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1. Let $M = \mathsf{H}^3/G$ be as in the introduction, and let γ be a closed geodesic in M. A lift $\tilde{\gamma}$ of γ in H^3 is then represented as the axis of a loxodromic isometry of H^3, which in turn is given by an element A in $PSL(2, \mathsf{C})$. The choice of a different lift $\tilde{\gamma}'$ gives rise to an element B in $PSL(2, \mathsf{C})$, such that A and B are conjugate in $PSL(2, \mathsf{C})$. A and B generate a subgroup of G, and hence a discrete subgroup of $PSL(2, \mathsf{C})$. If r denotes the infimum of the distance between $\tilde{\gamma}$ and $\tilde{\gamma}'$, as $\tilde{\gamma}'$ runs through all the lifts of γ different from $\tilde{\gamma}$, then the tubular neighborhoods of radius $r/2$ about all the lifts $\tilde{\gamma}'$ are all disjoint, and so project

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