Vol. 49, No. 1 DUKE MATHEMATICAL JOURNAL © March 1982

INTEGRABLE SYSTEMS WITH DELTA-POTENTIAL

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Dedicated to the dear memory of F. A. Berezin

1. Introduction. Let V be a Euclidean space of dimension d and let Δ be the Laplace operator in V. If $x_1, \dots, x_d$ are Euclidean coordinates in V, then $\Delta = \partial/\partial x_1^2 + \cdots + \partial/\partial x_d^2$. Let R be a locally finite set of (affine) hyperplanes in V (i.e., any bounded set intersects with a finite number of hyperplanes). Assume that for any $h \in R$ a real number $c(h)$ is given and denote by $\delta(h)$ the delta-function supported on h. By a system with δ-potential we mean a quantum system described by the Hamiltonian $H = \Delta - 2\sum_{h \in R} c(h)\delta(h)$. The δ-function $v = 2\sum_{h \in R} c(h)\delta(h)$ is the potential of our system. In spite of the singularity of v it is possible to interpret H as a self-adjoint operator in $L_2(V)$, but we will not go into it here. Instead we will be concerned with the eigenfunctions of H in the sense of distributions. This means that we investigate distributions satisfying the stationary Schrödinger equation $(H - E)f = 0$.

We use a well-known recipe for solutions of this equation (see the proof of Theorems 2.7 and 3.3 below for a justification). By a chamber in V we mean a closed polyhedron C formed by hyperplanes $h \in R$ and not containing inside itself any hyperplane from R. Denote by CB^∞ the space of continuous functions in V which are C^∞ in any chamber. For any $f \in CB^\infty$ and any $h \in R$ the jump of the normal derivative of f across h is well defined and we denote it by $j(\partial/\partial n)f$. The solutions of $(H - E)f = 0$ are functions $f \in CB^\infty$ solving the equation $(\Delta - E)f = 0$ on $V \setminus \bigcup_{h \in R} h$ and satisfying the condition

$$j(\partial/\partial n)f = 2c(h)f \tag{1.1}$$

on any $h \in R$. We will often refer to (1.1) as the cusp condition.

In this paper we will give a description of the eigenfunctions of H in the case when R is the set of reflecting hyperplanes of a discrete group generated by reflections. We need to establish some notation. Let s_h be the orthogonal reflection in V with respect to a hyperplane h; s_h belongs to the group $O(V)$ of metric preserving automorphisms of V. Discrete groups $G \subset O(V)$ generated by reflections s_h are called reflection groups. These groups have been classified by E. Cartan and Coxeter. We refer the reader to [1] and [2] for a detailed exposition and only briefly recall the results. First of all any reflection group can be decomposed into the product of irreducible reflection groups, so it suffices to consider those. Any irreducible finite reflection group is isomorphic to one of the

Received December 29, 1980. Work partially supported by NSF Grant MCS-7908621.