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ON THE SOLVABILITY OF GOURSAT PROBLEMS AND A FUNCTION OF NUMBER THEORY

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1. Introduction. In 1974, J. Leray showed an interesting example of Goursat problems (cf. [3], [4]):

$$\begin{cases} \epsilon u_{x_1x_2} = u_{x_1x_1} + u_{x_2x_2} + h(x_1,x_2), & (x_1,x_2) \in C^2 \\ u = O(x_1x_2) \end{cases} \tag{1.1}$$

where $u = O(x_1x_2)$ means that the function u/x_1x_2 is analytic at the origin. He proved that in case $\epsilon \in [-2,2]$ the solvability and uniqueness of (1.1) is closely connected with the algebraic-transcendental property of the parameter ϵ.

The purpose of this paper is, first to extend his results to a wider class of equations; secondly to show the close connection between the solvability and uniqueness of Goursat problems and the algebraic-transcendental properties of characteristic roots.

In §2 and §3 we shall prove the existence-and-uniqueness theorem under certain symmetry conditions of an equation (cf. (2.2), (2.3)), which contains the results of Leray's mentioned above as a special case. Moreover we obtain the eigenfunction expansion of the solution.

In §4 and §5 we study simple equations which do not necessarily satisfy the symmetry condition (2.2) or (2.3). We then introduce a function $\rho_\eta(\theta_1,\theta_2)$ which is a natural extension of Leray's auxiliary function ρ defined in [3] and that describes the transcendency of θ_1 and θ_2. In terms of this function we shall characterize the range of certain integro-differential operators corresponding to Goursat problems.

In the last section we consider a simple nonlinear Goursat problem and illustrate how the results in §2–§5 are connected with general theory of nonlinear Goursat problems.

2. Goursat problems for Hermitian type equations. Let $x = (x_1, \ldots, x_d)$ be the variable in the complex d-dimensional space C^d with $d \geqslant 2$. We use a multi-index $\alpha = (\alpha_1, \ldots, \alpha_d) \in Z^d$ with $|\alpha| = \alpha_1 + \cdots + \alpha_d$, where Z denotes the set of integers. We say that the multi-index α is nonnegative if all the components are nonnegative and write $\alpha \geqslant 0$. For a nonnegative multi-index α we define x^α by $x^\alpha = x_1^{\alpha_1} \ldots x_d^{\alpha_d}$. We denote by $(\partial/\partial x_\nu)^{-1}$ the integration with

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