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THE CLASSIFICATION OF COMPLETE MINIMAL SURFACES IN R^3 WITH TOTAL CURVATURE GREATER THAN -8π

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Introduction. One of the fundamental problems in the classical theory of minimal surfaces is the existence of complete immersed minimal surfaces in three dimensional Euclidean space. Two famous examples of these surfaces are the catenoid and Enneper's surface each of which has total Gaussian curvature equal to -4π. The complete minimal surfaces of finite total curvature have some very special analytic and geometric properties that are not shared by general minimal surfaces. For example, R. Osserman [9] has shown that if the total curvature of a complete minimal surface $f: M \to \mathsf{R}^3$ is finite, then M is "conformally" diffeomorphic to a compact "Riemann surface" $\overline{M}$ punctured in a finite number of points and the Gauss map $g: M \to \mathsf{P}^2$ extends "conformally" to $\overline{M}$. The reason for the quotes is that one needs to make sense of "conformal structure" and "conformal map" when a surface is nonorientable. An immediate corollary to Osserman's theorem is that the total curvature for a complete minimal surface in R^3 is always a multiple of -2π. R. Osserman then used the special properties of the Gauss map to prove that the plane is the unique complete minimal surface with total curvature greater than -4π and that the catenoid and Enneper's surface are the unique such orientable surfaces with total curvature -4π.

In this paper we present an elementary global analysis of the topology of complete nonorientable minimal surfaces $f: \overline{M} - \{p_1, \ldots, p_k\} \to \mathsf{R}^3$ with finite total curvature where $\overline{M}$ is compact. In the first section we prove a formula that states that the total curvature of this surface divided by 2π is congruent modulo 2 to the Euler characteristic of $\overline{M}$. This formula together with some topological results in [4] are then applied to prove that the total curvature of this nonorientable surface must be less than -4π and that if the total curvature of the surface equals -6π, then the surface is "conformally" diffeomorphic to a projective plane punctured in one or two points.

In section 2 we derive a Weierstrass type representation to analytically present nonorientable minimal surfaces which are homeomorphic to subsets of the projective plane. Using this representation, it is shown that the oriented two-sheeted cover of a minimal Möbius strip (with or without boundary) has well-defined associate surfaces and hence it is not rigid. The existence of associate surfaces places strong restrictions on the coordinate functions of the minimal Möbius strip. In section 3, these restrictions on the coordinate functions of a minimal Möbius strip are exploited to prove there exists a unique complete minimal Möbius strip with total curvature -6π.

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