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BIRATIONAL MORPHISMS OF SMOOTH THREEFOLDS COLLAPSING THREE SURFACES TO A CURVE

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The factorization problem, which has been an open question for some forty years, asks whether any birational correspondence f between algebraic spaces or varieties can be "factored" by a sequence of blowings-up with nonsingular centers, followed by a sequence of blowings-down to nonsingular centers. We will work in the category of algebraic spaces, and we will treat only the characteristic 0 case, where Hironaka's resolution of singularities and the domination theorem [5] allow us to reduce to the case where $f: X \to X'$ is actually a morphism, collapsing a divisor with normal crossings to a point or a curve with normal crossings, and is an isomorphism elsewhere. The surface case is a well known result of Zariski. The case when D has one component was settled by Moishezon [6], two components for dimension 3 by Schaps [9], and dimension 4 by Teicher [10]. Both of these latter results presumed D' to be actually nonsingular. If D' is singular with normal crossings the fiber dimension is bounded by 1, and the result follows from Danilov [4]. In what follows, we will require that D' have normal crossings in the strong sense that there are local coordinates at each point, such that the components of D' are defined by their vanishing. The main result of the paper is the factorization theorem for up to three components collapsing down to D', which is a normally crossing curve, in dimension 3. One hopes eventually for a general induction step. However, up through at least five components we have examples indicating that there are essential new difficulties to be encountered at each stage. The passage from two components to three required new techniques to deal with "resolution of singularities" problems, components of higher order (obtained by blowing-up intersections of exceptional divisors), the nonprojective example of Hironaka, and a new type of example of a surface collapsing to a node.

The first draft of this three-component-to-curve case employed Mori's recent work on collapsability [7] and one or two of the Kulikov-type arguments used extensively by Crauder in [2] to prove the three-component-to-point case (to which this paper may be viewed as a complement). However, from "philosophical" considerations, an attempt was made, successfully, to eliminate these arguments from the final version, in the interest of proving as much as possible with arguments which have an immediate extension to dimension n. This paper,

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