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OSCILLATORY INTEGRALS WITH SINGULAR SYMBOLS

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1. Introduction. Let X be an n-dimensional compact manifold and let P_i, $i = 1, \ldots, r$, be a first order pseudodifferential operator on X with real principal symbol, p_i. Assume the equations

$$p_1(x,\zeta) = \cdots = p_r(x,\zeta) = 0$$

are the defining equations for a non-radial, co-isotropic submanifold, $\sum$, of codimension k in $T^*X - 0$. The main goal of this paper will be to construct global parametrices for pseudodifferential operators of the form

$$P = P_1^2 + \cdots + P_r^2 + \sum_{i=1}^{r} A_i P_i + B \tag{1.1}$$

the A_i's and B being pseudodifferential operators of order zero. Note that because of the form of (1.1) the Levi condition

$$\sigma_{\text{sub}}|_{\Sigma} = 0 \tag{1.2}$$

is satisfied.

One encounters operators of the form (1.1) in connection with some problems in integral geometry. In particular the problem of showing that an overdetermined integral transform has for its range the solution set of a system of pseudodifferential equations can be reduced to the problem of constructing a parametrix for an operator such as (1.1). (See [5].) In 6 of [5] a vague sketch is given of how to construct such a parametrix. The details will be given here.

P is elliptic away from $\sum$; so if E is such a parametrix its wave-front set should consist of: (a) points on the diagonal, Δ, in $(T^*X - 0) \times (T^*X - 0)$, and (b) points on the joint flow-out from $\Delta \cap \sum$ by the H_{p_i}'s. The sets, (a) and (b), are Lagrangian submanifolds of $(T^*X - 0) \times (T^*X - 0)$ which intersect cleanly in a codimension k subset. Therefore, it is natural to attempt to express the Schwartz kernel of E as an oscillatory integral associated with a system of "paired Lagrangian manifolds" as in [7]. In [7] only the codimension one case (i.e., $k = 1$) was considered, whereas here we are concerned with arbitrary codimension; however, it will be helpful to review here some of the material in the first few sections of [7]. The distributions considered in [7] were microlocally modelled on

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