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SEMISTABLE DEGENERATIONS OF ENRIQUES' AND HYPERELLIPTIC SURFACES

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A degeneration of algebraic varieties (over the complex numbers) is a proper holomorphic map $\pi : X \to \Delta$ from a complex manifold to a disk, such that the fibres $X_t = \pi^{-1}(t)$ are smooth algebraic varieties for $t \neq 0$. The systematic study of degenerations of algebraic surfaces was begun by Persson [P], using the Clemens-Schmid exact sequence [C]. One case that is particularly relevant to the study of three-dimensional varieties is that of degenerations of algebraic surfaces of Kodaira number zero. These are abelian surfaces, $K3$ surfaces, Enriques' surfaces, and hyperelliptic surfaces.

Based on an earlier attempt of Kulikov [K], Persson and Pinkham [PP] have recently achieved a classification of semistable degenerations of $K3$ surfaces and abelian surfaces. This paper is devoted to semistable degenerations of Enriques' and hyperelliptic surfaces with trivial bicanonical bundle, and we obtain a complete classification (see Corollaries 6.2 and 6.3 for the precise statements). Since after base-change every degeneration is birational to a semistable one, this result provides a good beginning for the study of arbitrary degenerations (and is in fact enough for many applications). Non-semistable degenerations have been studied by Ueno [U] in the case of abelian surfaces, and by Nikulin [Ni] in the case of $K3$ surfaces.

As an application we prove a theorem stated by Horikawa [H], to the effect that the image of the period map for Enriques' surfaces is the complement of a union of certain specific hypersurfaces in period space. Horikawa's proof appealed to the Borel extension theorem [B] which requires that the period map be locally liftable; however, it is not clear that this is the case for the Enriques' period map. We deduce the theorem from our classification, which includes a characterization of the degenerations of Enriques' surfaces in terms of the monodromy action on the covering family of $K3$ surfaces.

The plan of the paper is as follows: in Section 1 we collect the known facts about degeneration of surfaces with Kodaira number zero, and in Section 2 review the technique of "generic contraction" introduced by Persson and Pinkham [PP]. Sections 3–5 are devoted to a classification theorem for degenerations of surfaces with $2K \equiv 0$, and in Section 6 we refine this somewhat in the cases of Enriques' and hyperelliptic surfaces. (Degenerations of the

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