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PROPER HOLOMORPHIC MAPPINGS AND THE BERGMAN PROJECTION

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Suppose that D_1 and D_2 are two smooth bounded pseudoconvex domains contained in $\mathbf{C}^n$, and suppose that f is a proper holomorphic mapping of D_1 onto D_2. In this note, we prove that if the $\bar{\partial}$-Neumann problem for D_1 satisfies global regularity estimates, then $u = \mathrm{Det}[f']$ extends smoothly to bD_1 and f extends smoothly to $bD_1 - \{z : u(z) = 0\}$. It is a classical result that f is a branched cover of some finite order m. Hence, we can define m local inverses $F_1, F_2, \ldots, F_m$ to f away from $\{f(z) : u(z) = 0\}$. We prove that if h is a holomorphic function in $C^\infty(\overline{D_1})$, then any symmetric function of $(h \circ F_1), (h \circ F_2), \ldots, (h \circ F_m)$ is a holomorphic function on D_2 which is in $C^\infty(\overline{D_2})$.

Let us say that a variety $V = \{h_1 = h_2 = \cdots = h_r = 0\}$ is smooth up to the boundary of D if the functions $h_1, h_2, \ldots, h_r$ are holomorphic on D and smooth up to the boundary of D. The result on the symmetric functions of $F_1, F_2, \ldots, F_m$ allows us to prove that if $V = \{h_1 = 0\}$ is a variety in D_1 which is smooth up to the boundary, then $f(V)$ is a variety in D_2 which is smooth up to the boundary. In particular, the variety $\{f(z) : u(z) = 0\}$ is smooth up to bD_2.

We also prove that the graph of f can be realized as a variety in $D_1 \times D_2$ which is smooth up to the boundary.

The proofs of the facts above rely on the observation that the same transformation rule which holds for the Bergman projections under biholomorphic mappings also holds for proper mappings. This allows techniques used in [2] to be applied to proper mappings.

The transformation rule for the Bergman projections under proper holomorphic mappings implies a corresponding transformation rule for the Bergman kernel functions.

The letter C will never denote the same constant twice in this paper. If D is a bounded domain contained in $\mathbf{C}^n$, the Bergman projection associated to D is the orthogonal projection of $L^2(D)$ onto its subspace consisting of holomorphic functions.

THEOREM 1. *If P_i denotes the Bergman projection associated to the bounded domain D_i contained in $\mathbf{C}^n$, $i = 1, 2$, and f is a proper holomorphic mapping of D_1 onto D_2, then*

$$P_1(u \cdot (\phi \circ f)) = u \cdot ((P_2 \phi) \circ f)$$

where $u = \mathrm{Det}[f']$ and $\phi \in L^2(D_2)$.

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