Vol. 48, No. 1 DUKE MATHEMATICAL JOURNAL © March 1981

GRIFFITHS' INTEGRAL FORMULA FOR THE MILNOR NUMBER

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Introduction. In [1], Griffiths obtained an integral formula for the Milnor number μ of an isolated hypersurface singularity:

$$(-1)^n \mu + 1 = \lim_{\epsilon \to 0} \lim_{t \to 0} \left[\sum_{k=0}^{n} \frac{C(k,n)}{\epsilon^{2k}} \int_{V_t[\epsilon]} c_{n-k}(V_t) \wedge \phi^k \right]. \tag{1}$$

Here V_t is one of a family of complex analytic hypersurfaces in $\mathbf{C}^{n+1}$ acquiring an isolated singularity at $t = 0$, $V_t[\epsilon]$ is the intersection of V_t with a ball of radius ϵ centered at the singularity, $c_{n-k}(V_t)$ is the $(n-k)$th Chern form in the curvature of V_t, ϕ is the standard Kähler form on $\mathbf{C}^{n+1}$, and $C(k,n)$ is a constant depending only on k and n. (Griffiths did not calculate the $C(k,n)$ but he did show that they are positive.)

Formula (1) resembles the higher-dimensional Gauss–Bonnet formula [2]. Indeed, the left side of (1) is the Euler characteristic of the Milnor fiber $V_t[\epsilon]$, while the leading term on the right is basically the same as the integral occurring in Gauss–Bonnet.

This note will show that in fact the Euler characteristic χ of the Milnor fiber is given by

$$\chi = \lim_{\epsilon \to 0} \lim_{t \to 0} C(n) \int_{V_t[\epsilon]} c_n(E_t), \tag{2}$$

where E_t is a "twisted tangent bundle" obtained by tensoring the tangent bundle of V_t with a certain line bundle, and where $C(n)$ is the same constant which appears in the Gauss–Bonnet formula. Using formula (2), one can readily obtain Griffiths' formula, with the constants explicitly determined.

Section 1 states (2) precisely. Sections 2 and 3 are devoted to its proof, section 4 to Griffiths' formula.

For definitions of the Milnor fiber and Milnor number, see [3]. For the fundamentals of curvature and characteristic classes, see [4] or [5]. Griffiths' article [1] explains the notion of "defect" and establishes certain notations which are copied here.

Note in particular that the phrase "analytic variety" does not mean "analytic manifold".

Received November 17, 1980.