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OBSTRUCTIONS TO MODULAR CLASSICAL SIMPLE LIE ALGEBRAS

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Introduction. Let $\mathfrak{M}$ be a finite-dimensional simple Lie algebra over C, the field of complex numbers. The usual classification theory associates to $\mathfrak{M}$ an integer l (the rank of $\mathfrak{M}$) and an $l \times l$ integral matrix $A = (A_{ij})$ (the Cartan matrix of $\mathfrak{M}$). Let $X = \{e_i, f_i, h_i | 1 \leqslant i \leqslant l\}$ and $\mathfrak{I}$ be the ideal of $\mathfrak{FL}(X)$ (the free Lie algebra on X) generated by the elements

$$\begin{cases} [e_i, f_j] - \delta_{ij} h_i, \\ [h_i, e_j] - A_{ij} e_j, \\ [h_i, f_j] + A_{ij} f_j, \\ [h_i, h_j], \textit{for } 1 \leqslant i, j \leqslant l. \end{cases} \tag{0.1}$$

Let $\tilde{\mathfrak{L}} = \tilde{\mathfrak{L}}(A)$ denote $\mathfrak{FL}(X)/\mathfrak{I}$. Then one has that $\mathfrak{M}$ is a homomorphic image of $\tilde{\mathfrak{L}}$. The usual isomorphism theorem (e.g. [5, Theorem 4.3]) for finite-dimensional simple Lie algebras over C states that if $\mathfrak{N}$ is any finite-dimensional simple homomorphic image of $\mathfrak{L}(A)$ then $\mathfrak{M} \cong \mathfrak{N}$. In fact, more is true. By [1, Theorem 1.10] $\tilde{\mathfrak{L}}(A)$ contains a unique maximal ideal $\mathfrak{R}$. Serre's Theorem [15, P. VI–19], [4, Theorem 18.3] shows that $\mathfrak{R}$ is generated by the images in $\tilde{\mathfrak{L}}(A)$ of the elements

$$(ad e_i)^{-A_{ij}+1} e_j, \ 1 \leqslant i \neq j \leqslant l, \tag{0.2}$$

and

$$(ad f_i)^{-A_{ij}+1} f_j, \ 1 \leqslant i \neq j \leqslant l. \tag{0.3}$$

Serre also proves [15, P. VI–26] that $\mathfrak{R}$ is the unique proper ideal of finite codimension in $\tilde{\mathfrak{L}}$.

In this paper we investigate whether analogous results hold over fields of prime characteristic. Thus, we let F be an algebraically closed field of characteristic $p > 0$ and let $\mathfrak{M}$ be a classical (in the sense of [14, P. 28]) simple modular Lie algebra over F. (Note that this forces $p > 3$). The Mills-Seligman classification

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