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SUBELLIPTIC ESTIMATE FOR THE $\bar{\partial}$-NEUMANN PROBLEM

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Introduction. This article is devoted to the proof of a sufficient condition for subellipticity of the $\bar{\partial}$-Neumann problem in an open set Ω, at points having a neighborhood where the boundary is pseudoconcave and real-analytic (Theorem 3.1).

As in the pseudoconvex case ([K], Th. 1.19) our hypothesis is purely geometrical. We first prove a sufficient condition in terms of partial holomorphic dimension (the notion of holomorphic dimension was introduced by J. J. Kohn in [K], and the notion of partial holomorphic dimension appears implicitly in a previous work of one of the authors [D]). This is done in theorem 6.1.

In a second step, we show that the hypothesis (H) of theorem 6.1 is equivalent to a more natural geometrical hypothesis (Theorem 7.1).

For example, we deduce the following: let Ω_1, Ω_2 be two bounded pseudoconvex domains in $\mathbf{C}^n$ with real analytic boundary such that $\bar{\Omega}_1 \subset \Omega_2$. Let $\Omega = \Omega_2 \backslash \bar{\Omega}_1$. Then the $\bar{\partial}$-Neumann problem in Ω is subelliptic for (p,q) forms at every point of $\partial\Omega$ if and only if $1 \leqslant q \leqslant n-2$.

In the proofs, we consider only the case $p = 0$, because p plays only the role of a parameter, and we restrict ourselves to open sets of $\mathbf{C}^n$, but our results (which are local) work equally well in domains on complex manifolds.

2. Definitions and notations. Let Ω, open in $\mathbf{C}^n$, be defined, near $0 \in \partial\Omega$, by a function $r(C^\infty$ or real-analytic; this means $\Omega = \{Z | r(Z) < 0\}$ $dr \neq 0$ on $\{r = 0\}$. This is specified in the theorems). We consider n vector fields $L_1, \ldots, L_n$ near 0, such that:

(i) $L_1, \ldots, L_{n-1}$ are tangential to $\partial\Omega$, $L_i,(r) = 0$ on $\partial\Omega i = 1, \ldots, n-1$.

(ii) L_n is transverse to $\partial\Omega$ (let $L_n(r) = 1$ on $\partial\Omega$).

Let $(\omega_1, \ldots, \omega_n)$ be the dual system of (1,0) forms. Then every $(0,q)$ form can be written in the form

$$\varphi = \sum_{|I|=q} \varphi_I \bar{\omega}_I$$

where $I = (i_1, \ldots, i_q)$, $\bar{\omega}_I = \bar{\omega}_{I_1} \wedge \ldots \wedge \bar{\omega}_{i_q}$, $i_1 < \ldots < i_q$.

Let J be a multi-index; if I is a multi-index obtained by removing index i from

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