Vol. 48, No. 1 DUKE MATHEMATICAL JOURNAL © March 1981

THE QUASI-CLASSICAL LIMIT OF QUANTUM SCATTERING THEORY II, LONG-RANGE SCATTERING

KENJI YAJIMA

0. Introduction. In our previous paper [16], which will be referred to as [I] hereafter, we studied the quasi-classical limit of the scattering operator for Schrödinger operator with short range potentials. In this paper, we extend the results of [I] to the case where the potential is long range.

We consider the Schrödinger equation

$$i\hbar(\partial u/\partial t)(t,x) = -(\hbar^2/2m)\Delta u(t,x) + V(x)u(t,x) \qquad \text{in } \mathfrak{H} = L^2(R^n) \quad (0.1)$$

and the corresponding Hamilton equation

$$m\dot{x}(t) = \xi(t), \qquad \dot{\xi}(t) = -\operatorname{grad}_x V(x(t)), \qquad (x(t),\xi(t)) \in R^n \times R^n, \quad (0.2)$$

where $\Delta = \sum_{j=1}^n \partial^2/\partial x_j^2$, $m > 0$, $\hbar$ is Planck's constant divided by 2π and the dots stand for the derivative with respect to the variable t. We assume that the potential satisfies the following assumption.

Assumption (AL). $V(x)$ is a real valued C^∞-function on R^n and there exists a constant $\epsilon > 0$ such that for $m = 0, 1, \ldots,$

$$V_m \equiv \max_{|\alpha| = m} \sup_{x \in R^n} |(\partial/\partial x)^\alpha V(x)| \cdot (1 + |x|)^{m+\epsilon} < \infty. \quad (0.3)$$

We write $H_0^h \equiv -(\hbar^2/2m)\Delta$, $H^h \equiv H_0^h + V(x)$ and $H(x,\xi) \equiv \xi^2/2m + V(x)$. As was first pointed out by Dollard [4], under Assumption (AL), the wave operator

$$W_\pm^h \equiv s - \lim_{t \to \pm\infty} \exp(itH^h/\hbar)\exp(-itH_0^h/\hbar) \quad (0.4)$$

does not in general exist and to develop a suitable scattering theory one must introduce a modified free propagator. After Dollard [4], scattering theory for long range potentials was studied by many authors [1], [3], [6], [7], [8], [9], [10], [14], [17] and [18], and the question of the completeness of the modified wave operators was eventually settled by [9], [10], [17] and [18] in the quantum case and by [6] in the classical case. We record here the main result of the theory both in the quantum case and the classical case. For a function $f(x)$, $\partial f/\partial x$ is its

Received May 27, 1980. Revision received November 21, 1980.