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GENERALIZATIONS OF LIPSCHITZ SPACES AND AN APPLICATION TO HARDY SPACES AND BOUNDED MEAN OSCILLATION

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Introduction. Lipschitz or Besov spaces and generalizations of them have been introduced and studied by many authors: e.g., Zygmund [33], Taibleson [27], Caldéron [2], Herz [11], Peetre [18], Triebel [30], to mention a few. Two such generalizations, more or less overlapping with the works of these authors, are defined and studied here. One (denoted by $\mathrm{Lip}(\rho, E)$ or $\Lambda(\rho, E)$) includes the Besov spaces $B_p^{s\infty}$ for $s > 0$ (the Lipschitz spaces) and the other (denoted by $B(\rho, E)$) includes B_p^{s1} for $s < 0$. The important feature of this paper is that the function ρ used in the definitions may grow arbitrarily slowly.

Section 5 gives a more general definition, including all Besov spaces $B_p^{sq}, s \neq 0$ ($p, q \geqslant 1$, since only Banach spaces are considered). There is also given a link between the two types of spaces.

Section 1 covers some preliminaries.

Section 2 gives the basic definitions. $\mathrm{Lip}(\rho, E)$ is the space of distributions whose moduli of continuity in the Banach space E are dominated by the rather arbitrary function ρ. $B(\rho, E)$ is defined using convolutions with test functions, cf. Flett [7] and Taibleson [27]. Equivalent definitions of the spaces are given in later sections.

Section 3 introduces a modification of $\mathrm{Lip}(\rho, E)$ denoted by $\Lambda(\rho, E)$. It may be defined using higher differences, so we have the same situation as classically with the Lipschitz space Lip_1 and the Zygmund class Λ_*. The two spaces coincide when ρ grows slowly.

Section 4 gives duality theorems.

Section 5 was described above.

Section 6 and 7 treat multipliers and interpolation, without going into details.

Section 8 is an application to Hardy spaces. It is shown that $BMO(\rho)$ is the dual space of H_ω, for suitable pairs of functions ρ and ω (Theorem 13). The proof of this duality was in fact the starting point from which the present paper grew out.

We are so far dealing with functions and distributions on R^d, but everything holds also (with a few simplifications) on the torus T^d. In fact, many of the references given in the text treat only this case.

Section 9 gives more details on this case, and on the connection with spaces of

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