Vol. 47, No. 4 DUKE MATHEMATICAL JOURNAL © December 1980

THE MATHEMATICAL THEORY OF RESONANCES WHOSE WIDTHS ARE EXPONENTIALLY SMALL

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§1. **Introduction.** Among the most interesting and challenging mathematical problems raised by non-relativistic quantum mechanics are those connected with resonances. This is due both to their subtle nature and to their practical importance. Let us begin by describing two situations which are typical of rather distinct general classes of examples; the first was completely analyzed by one of us [49] several years ago (relying, in part, on earlier work of Friedrichs and Howland; see [49] for references)—the second will be analyzed in this paper.

Example 0. Autoionizing states in helium. Let A_0 be the Schrödinger operator

$$A_0 = -\Delta_1 - \Delta_2 - \frac{2}{r_1} - \frac{2}{r_2}$$

on $L^2(\mathsf{R}^6)$, where $\mathbf{r}$ in R^6 is written $\langle \mathbf{r}_1, \mathbf{r}_2 \rangle$, and Δ_i is the Laplacean in $\mathbf{r}_i$. Let

$$A(\lambda) = A_0 + \lambda |\mathbf{r}_1 - \mathbf{r}_2|^{-1}$$

(A_0 and $A(\lambda)$ are self-adjoint with domains taken as the Sobolev space H_2). For $\lambda = 1$, A_0 is a very good approximation to the energy operator of the helium atom (and for $\lambda = (2/Z)^2$, to a constant multiple of the energy operator of an ion consisting of a nucleus of charge Z and two electrons). The continuous spectrum

Received March 29, 1980. Research partially supported by USNSF grant MCS 78-01885. Part of this work was done while E. Harrell held a NSF National Needs Fellowship at the Department of Mathematics, M.I.T.