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GALOIS CONJUGACY OF UNRAMIFIED TWISTS OF HECKE CHARACTERS

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The purpose of this paper is to prove two theorems of a purely algebraic nature which find applications in the theory of Hecke L-functions (cf. [3], Prop. 2). The phenomenon described in the title has been noted previously, cf. Gross [2].

§1. We assume that for each algebraic number field L we are given an abelian group H_L and for each pair of number fields K, L with $K \supset L$ a group homomorphism

$$h_{K/L} : H_L \to H_K$$

such that the following conditions are satisfied:

(i) $h_{L/L} = \mathrm{id}$.

(ii) Given $K \supset M \supset L$ we have $h_{K/L} = h_{K/M} \circ h_{M/L}$.

(iii) The degree $[K : L]$ is an exponent for the kernel of $h_{K/L}$.

Examples

(1) Let L^* denote the multiplicative group of L and L^{*n} the subgroup of nth powers. Let $H_L = L^*/L^{*n}$ and let

$$h_{K/L} : L^*/L^{*n} \to K^*/K^{*n}$$

be the natural map. Then conditions (i), (ii), and (iii) are satisfied. Indeed (iii) follows from the fact that if

$$N_{K/L} : K^* \to L^*$$

is the norm and

$$\mathrm{inc}_{K/L} : L^* \to K^*$$

is inclusion, then the composition $N_{K/L} \circ \mathrm{inc}_{K/L}$ has the form $\alpha \to \alpha^{[K:L]}$.

(2) Let μ_L denote the group of roots of unity in L. Then conditions (i), (ii), and (iii) are satisfied with $H_L = L^*/\mu_L L^{*n}$ and

$$h_{K/L} : L^*/\mu_L L^{*n} \to K^*/\mu_K K^{*n}$$

the natural map.

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