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A SELF-AVOIDING RANDOM WALK

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1. Introduction. Let $\mathbf{Z}^d = \{(z_1, \ldots, z_d) : z_i \in \mathbf{Z}\}$ be the integer lattice. A k-step self-avoiding path is a sequence of points $[x_0, x_1, \ldots, x_k]$ in $\mathbf{Z}^d$ with $x_0 = 0$, $\|x_i - x_{i-1}\| = 1$ for $i = 1, \ldots, k$, and $x_i \neq x_j$ for $i \neq j$. The study of self-avoiding paths originated in chemical physics as an attempt to model polymer chains. Two major questions were asked. First, if $\gamma_d(k)$ is the number of self-avoiding paths of length k, then how does $\gamma_d(k)$ grow with k? Second, what is the mean square distance of the self-avoiding walk from the origin, or more generally, what is the limiting distribution as k approaches infinity of the walk after k steps? While much numerical evidence has been gathered to answer these questions, especially in dimensions two and three, few sharp analytical results have been found. For a review of the work, both analytical and numerical, see [3].

Hammersley [12] first noted the relation $\gamma_d(j)\gamma_d(k) \geqslant \gamma_d(j + k)$ which implies that $\log \gamma_d(k)$ is a subadditive function of k. Hence there exists a constant β_d called the connective constant such that

$$\lim_{k \to \infty} (\gamma_d(k))^{1/k} = \beta_d.$$

Kesten [8] showed that

$$\lim_{k \to \infty} \gamma_d(k + 2)/\gamma_d(k) = \beta_d^2,$$

but whether $\lim_{k \to \infty} \gamma_d(k + 1)/\gamma_d(k)$ exists is still an open question.

Let $S(n)$ be simple or unrestricted random walk on $\mathbf{Z}^d$. The distribution given to self-avoiding paths of length k was the conditional distribution of the

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