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CURVATURE CHARACTERIZATION OF HYPERQUADRICS

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After proving the dimension two case jointly with Andreotti, Frankel [3] conjectured that a compact Kähler manifold of positive sectional curvature is biholomorphic to the complex projective space. Mabuchi [8] verified the case of dimension three by using the result of Kobayashi-Ochiai [6]. Very recently by using the methods of algebraic geometry of positive characteristic Mori [10] proved that a compact Kähler manifold with ample tangent bundle must be biholomorphic to the complex projective space. By methods of Kähler geometry Siu-Yau [12] proved that a compact Kähler manifold of positive holomorphic bisectional curvature must be biholomorphic to the complex projective space. Frankel's conjecture is a special case of these more general results. It is reasonable to conjecture that there are similar curvature characterizations for other irreducible compact symmetric Kähler manifolds. In this paper we obtain such a curvature characterization for the complex hyperquadric.

Definition. Let M be a Kähler manifold and $P \in M$. If the holomorphic bisectional curvature of M is nonnegative at P, then for a nonzero element $\xi = (\xi^\alpha)$ of the holomorphic tangent space $T_P M$ of M at P, the *curvature null space* at P in the direction of ξ, denoted by $N_P(\xi)$, is defined as the set of all $\eta = (\eta^\alpha) \in T_P M$ such that

$$\sum R_{\alpha\bar{\beta}\gamma\bar{\delta}} \xi^\alpha \overline{\xi^\beta} \eta^\gamma \overline{\eta^\delta} = 0$$

where $R_{\alpha\bar{\beta}\gamma\bar{\delta}}$ is the curvature tensor of M. Due to the nonnegativity of the holomorphic bisectional curvature at P, $N_P(\xi)$ is a complex linear subspace of $T_P M$. The curvature tensor of M is said to be *m-positive* at P if the holomorphic bisectional curvature of M is nonnegative at P and if $\dim_{\mathbb{C}} N_P(\xi) < m$ for every $0 \neq \xi \in T_P M$.

Note that 1-positivity is equivalent to the positivity of the holomorphic bisectional curvature.

The curvature characterization of the hyperquadric is given by the following.

MAIN THEOREM. *Let $n \geqslant 3$ and let M be a compact Kähler manifold of complex dimension n. Suppose the curvature tensor of M is m-positive everywhere for some $m < n/2 + 1$ and is 2-positive at some point of M. Then M is biholomorphic to either the complex projective space or the complex hyperquadric.*

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