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NOTE ON A PAPER OF F. TREVES CONCERNING MIZOHATA TYPE OPERATORS

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0. Introduction. To simplify notations we shall systematically work with germs of smooth functions at the origin in R^2 (or C). Thus $C^\infty(\mathsf{R}^2)$ stands for germs of smooth functions, and similarly for functions defined in a half-plane.

In [2] Treves investigates Mizohata type vector fields in R^2, which by definition are (germs of) smooth complex vector fields L defined near 0 such that

$$L(0), \overline{L(0)} \text{ are } \mathsf{C}\text{-linearly dependent} \tag{0.1}$$

$$L(0), \left[L(0), \overline{L(0)} \right] \text{ are } \mathsf{C}\text{-linearly independent.} \tag{0.2}$$

It is then easy, after a change of variables and multiplication to the left by a non-vanishing (germ of smooth) function to reduce L to the form,

$$L = \frac{\partial}{\partial t} - i t\, g(t,x) \frac{\partial}{\partial x}, \quad \operatorname{Re} g(0,0) \neq 0. \tag{0.3}$$

Treves shows that after a further change of variables conserving $t \leqslant 0$ and $t \geqslant 0$ and multiplication by a non-vanishing factor, L can be reduced to

$$L = \frac{\partial}{\partial t} - i t(1 + \rho(t,x)) \frac{\partial}{\partial x}, \tag{0.4}$$

where ρ vanishes to infinite order on $t = 0$. The argument is simple; suppose for simplicity that $\operatorname{Re} g(0,0) > 0$ (which can be achieved after the change $x \to -x$). Then by Taylor expanding in t we can construct u with the Taylor series

$$u(t,x) \sim x + \frac{it^2}{2} g(0,x) + a_3(x)t^3 + \cdots \tag{0.5}$$

such that Lu vanishes to infinite order on $t = 0$: $Lu = O(t^\infty)$, and there is a change of variables $\tilde{x} = x(t,x)$, $\tilde{t} = \tilde{t}(t,x)$ conserving $t \geqslant 0$, $t \leqslant 0$ so that

$$u(t,x) = \tilde{x} + \frac{i\tilde{t}^{\,2}}{2}. \tag{0.6}$$

In the new coordinates L will necessarily take the form (0.4). When $\rho = 0$, L is the Mizohata operator.

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