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WATER WAVES GENERATED BY A PRESSURE DISTURBANCE ON A STEADY STREAM

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1. Introduction. The purpose of this paper is to develop a method for constructing steady flows of water resulting from a localized pressure disturbance on the surface. The flows considered are exact solutions of the equations satisfied by an incompressible, inviscid fluid bounded above by a free surface. The flow is determined by the pressure at the surface and the effect of gravity and surface tension. The pressure disturbance is assumed small, and the flows obtained are perturbations of a uniform horizontal flow of speed U_0 and constant depth h. If U_0 is small, the perturbed flow tends to the uniform flow at infinity; however, if U_0 is larger than a certain value, a wave pattern appears. Two-dimensional flows which exhibit periodic behavior at infinity are constructed here, as well as three-dimensional flows which become uniform at infinity. For three-dimensional flows with U_0 sufficiently large, patterns like the wake behind a ship should be created; it would undoubtedly be much more difficult to find exact solutions in this case.

The problem is to determine the free surface and a velocity field in the associated domain satisfying the conditions of incompressibility and irrotationality, the streamline conditions on top and bottom, and Bernoulli's equation on the top surface; see (2.1)–(2.5) for the two-dimensional case and Section 5 for three dimensions. If the pressure disturbance is $\epsilon p(X)$, we can approximate the change in the flow to first order in ϵ by imposing the top boundary conditions on the unperturbed flat surface rather than on the unknown perturbed surface. In the two-dimensional case, if the change in the free surface is written as $\epsilon h\eta(X)$, it is found that the first approximation to η must satisfy an equation of the form

$$b(\xi)\hat{\eta}(\xi) + \hat{p}(\xi) = 0, \tag{1.1}$$

where ˆ denotes the Fourier transform and

$$b(\xi) = 1 + \beta\xi^2 - \gamma\xi \coth \xi. \tag{1.2}$$

Here β is related to the coefficient of surface tension and $\gamma = U_0^2/gh$ is the Froude number. For U_0 small enough, b has no real zeros; thus if $p(X)$ is smooth and decays rapidly at infinity, the same is true of $\eta(X)$. However, if U_0

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