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A NOTE ON THE Γ-SPECTRUM OF AN AUTOMORPHISM GROUP

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Introduction. Determination of the quantity $\Gamma(\alpha)$ relative to a non-commutative dynamical system (R, G, α), G abelian, has been shown to be important in determining structural properties. First of all, if α is the modular group of a factor R, $\Gamma(\alpha)$ furnishes the Connes invariant $S(R)$ (for a general von Neumann algebra one needs $\Gamma_B(\alpha)$, see below). Secondly, recent work of Connes-Takesaki [4], Kishimoto-Takai [8], Olesen-Pedersen [11], show that the maximality of $\Gamma\alpha$ is related to whether or not the crossed product, or the fixed point algebra is a factor (or simple in the C^*-case). $\Gamma(\alpha)$ and $\Gamma_B(\alpha)$ also measure the innerness of α.

The spectrum of an automorphism group is ostensibly easier to calculate than α. We shall give conditions under which the spectrum and Γ coincide. Note also that $\Gamma(\alpha) = \mathrm{sp}(\alpha)$ for the system (R, G, α) entails that $\Gamma(\dot{\alpha}) = H$ where $(R, H \equiv G/\ker(\alpha), \dot{\alpha})$ has the obvious meaning.

The origin of results of this nature is the, then surprising, result of Størmer [13] (see also [1], [6], [9]).

In this paragraph we determine conditions under which the Borchers spectrum $\Gamma_B(\alpha)$ (and the Connes spectrum $\Gamma(\alpha)$) coincide with $\mathrm{sp}(\alpha)$ for a W^*-dynamical system. We first recall some definitions.

1. By a *W^*-dynamical system* we mean a triple (R, G, α) consisting of a von Neumann algebra R, a locally compact group G and a pointwise ultraweakly continuous action $\alpha : G \to \mathrm{Aut}(R)$.

2. If G is abelian the Borchers spectrum of α is by definition, [12],

$$\Gamma_B(\alpha) = \bigcap_e \mathrm{Sp}(\alpha^e).$$

Here the spectrum of an automorphism is defined as in [2, 3], and e is an arbitrary projection in the fixed point algebra, R^α, such that its central support, $c(e)$, $= I$. The notation α^e means $\alpha \mid eRe$.

The Borchers spectrum enjoys many properties in common with $\Gamma(\alpha)$ (recall that $\Gamma(\alpha)$ is defined requiring $e \neq 0$ instead of $c(e) = I$). Note that if α is centrally ergodic, then $\Gamma_B(\alpha) = \Gamma(\alpha)$ because for any projection $o \neq e \in R^\alpha, c(e)$ is α-invariant, thus $c(e) = I$.

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