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SINGULAR SOLUTIONS FOR ANALYTIC PSEUDODIFFERENTIAL OPERATORS OF PRINCIPAL TYPE

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0. Introduction and statements of results. We are interested in analytic pseudodifferential operators P. We wish to construct distributions u, so that Pu is analytic and u has prescribed wave front set. Our motivation, in part, comes from Baouendi-Treves-Zachmanoglou [1], who constructed solutions to analytic partial differential equations with prescribed zeroes or singular support. See also Duistermaat-Hörmander [12], Hörmander [13] and Sjöstrand [10] for some beautiful results in the C^∞ category. Of course in [12] and [13] condition (P) is assumed throughout, whereas in our present study condition (P) may be violated.

To be precise now, let P be a classical analytic pseudodifferential operator of degree m on an open subset $\Omega \subset \mathbf{R}^n$. (See for example [3], [5] for the theory.) Let p be its principal symbol and $\rho \in T^*(\Omega)\backslash 0$ a characteristic point. We will assume P to be of principal type at ρ, that is, we assume that for some complex number z, the Hamilton field of $\mathrm{Re}(zp)$, $H_{\mathrm{Re}(zp)}$ and the radial vector field, $\partial/\partial\lambda$ are linearly independent at ρ.

Consider the Lie algebra $\mathcal{Q}$ generated by $H_{\mathrm{Re}\ p}$, $H_{\mathrm{Im}\ p}$ and $\partial/\partial\lambda$. These three vector fields have analytic coefficients and hence determine a foliation of $T^*(\Omega)\backslash 0$ by Nagano's Theorem [8]. Note that the dimension of the leaves need not be constant as in the classical Frobenius theorem. Let $\mathcal{L}$ be the leaf of this foliation through ρ. It is an analytic conic submanifold of $T^*(\Omega)\backslash 0$, with tangent space at each point equal to $\mathcal{Q}$. Since P is of principal type we may write dimension $\mathcal{L} = k + 1$ with $k \geqslant 1$.

The general question we wish to consider is the following: When can we find $u \in \mathcal{D}'(\Omega)$ so that Pu is analytic and $WF(u) = \mathcal{L}$? Certainly a necessary condition for the above to hold is that

$$\mathcal{L} \subset \{\rho \in T^*(\Omega)\backslash 0 : p(\rho) = 0\} \tag{0.1}$$

Actually this is the only assumption we need. It is clearly invariant under canonical transformation and multiplication of P by an elliptic factor. The main result of this paper is

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