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# NOTE ON THE DECAY OF ACOUSTIC WAVES

JAMES RALSTON

In [6] Melrose uses the fundamental results of [7] to deduce uniform local decay theorems for the wave equation in "nontrapping" exterior domains. The purpose of this note is to point out that [7], combined with the results of Vainberg [12], implies slightly stronger theorems. This approach was suggested by J. Rauch in his extension of Vainberg's work, [9], and it leads to the estimate (6) below.

As in [6] we consider the following mixed problems in an exterior domain $\Omega \subset \mathsf{R}^n$ with smooth boundary $\partial\Omega$:

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = 0 \quad \text{on} \quad \mathsf{R} \times \Omega, \qquad u = 0 \quad \text{on} \quad \mathsf{R} \times \partial\Omega.$$

$$u(0,x) = 0, \qquad \frac{\partial u}{\partial t}(0,x) = f \tag{D}$$

and

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = 0 \quad \text{on} \quad \mathsf{R} \times \Omega, \qquad \frac{\partial u}{\partial \nu} = 0 \quad \text{on} \quad \mathsf{R} \times \partial\Omega$$

$$u(0,x) = 0, \qquad \frac{\partial u}{\partial t}(0,x) = f, \tag{N}$$

where the initial value $f$ is supported in $\Omega \cap |x| < R$. We assume throughout that $\Omega$ contains $|x| \geqslant R$.

If $\Omega$ is nontrapping in the sense made precise in [6], then the results of [7] imply that the fundamental solutions for the problems (D) and (N) have distribution kernels $E(t; x, y)$ which are actually smooth functions on the closed set

$$\{(x, y) \in \overline{\Omega} \times \overline{\Omega} : |y| \leqslant R \text{ and } |x| \leqslant t - T(R)\}.$$

Thus, defining $E_0 = \rho E$, where $\rho(t,x)$ is a smooth function on $\mathsf{R}_+ \times \overline{\Omega}$ such that

(i) $\rho(t,x) = h(t)$ on a neighborhood of $\partial\Omega$,

(ii) $\rho \equiv 1$ on $t - T(R) - 1 < |x| < t + R + 1$, and

(iii) $\rho \equiv 0$ on $|x| < t - T(R) - 2$ and $|x| > t + R + 2$,

one sees immediately that $E_0$ satisfies Vainberg's condition D′ with $N = \infty$. Thus, letting $R(k) = (k^2 + \Delta)^{-1}$, where $\operatorname{Im} k > 0$ and $\Delta$ denotes the Laplacian in

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