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THE EXTREME POINTS OF SOME CLASSES OF HOLOMORPHIC FUNCTIONS

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1. Introduction. Let $\overline{W}$ be a bordered Riemann surface (our notation and terminology is that of [3]). We will denote by $H(W)$ the class of all holomorphic functions on W, and we will denote by $N(W)$ the class of those f in $H(W)$ such that $\operatorname{Re} f > 0$. Let $\zeta \in W$ and let

$$N(W, \zeta) = \{f : f \in N(W), f(\zeta) = 1\}.$$

Thus $N(W, \zeta)$ is convex (and compact in the compact open topology). In this paper we find the extreme points of $N(W, \zeta)$ if $\overline{W}$ is compact and planar. This is Corollary 3.3 of section 3. Section 2 contains two propositions which will be used in section 3.

This paper might be thought of as an appendix to [7].

2. Functional analysis

2.1. Let X be a compact Hausdorff space and let

$$P(X) = \{\mu : \mu \in M_+(X), \mu(X) = 1\},$$

where by $M_+(X)$ we mean the class of all Radon measures on X. Thus if $\mu \in M_+(X)$ and $E \subset X$, then $\mu(E) \geqslant 0$. Let $\varphi_1, \ldots, \varphi_n \in C(X, \mathsf{R})$, let $\theta_1, \ldots, \theta_n \in \mathsf{R}$, and let

$$B = \left\{ \mu : \mu \in P(X), \int \varphi_j \, d\mu = \theta_j, \ 1 \leqslant j \leqslant n \right\}.$$

PROPOSITION. *Let $\mu \in \partial B$, where by ∂B we mean the class of all extreme points of B. Then μ is a convex combination of $n + 1$ members of $\partial P(X)$.*

Proof. We let $\varphi = (1, \varphi_1, \ldots, \varphi_n)$ and $\theta = (1, \theta_1, \ldots, \theta_n)$. Then

$$\int \varphi \, d\mu = \theta. \tag{2.1}$$

Let $E_1, \ldots, E_{n+2} \subset X$ be μ measurable sets such that (a) $E_j \cap E_k = \emptyset$ if $j \neq k$, and (b) $X = \bigcup_1^{n+2} E_j$. Let $\mu_j = \mu \llcorner E_j$. If $v_1, \ldots, v_{n+2}$ are vectors in R^{n+1},

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