Vol. 46, No. 4 DUKE MATHEMATICAL JOURNAL © December 1979

W^*-DYNAMICAL SYSTEMS SATISFYING A SPECTRUM CONDITION

JON KRAUS

One of the most fruitful areas of interaction between mathematics and physics in recent years has been the use of operator algebras in quantum field theory and statistical mechanics (see for example [5], [9], [10], [15], [23]). In particular, a number of mathematicians and physicists have been studying the properties of noncommutative dynamical systems (continuous actions of locally compact groups on operator algebras by *-automorphisms). In this paper we are concerned with a problem about W^*-dynamical systems which originated in quantum field theory. It can be roughly stated as: If the system satisfies a spectrum condition (with respect to a "positive" semigroup in the dual group), then is the action inner, and can the implementing unitary representation be chosen with positive spectrum?

We will now state the problem more precisely, and indicate its connection with quantum field theory. A *W^*-dynamical system* $\mathfrak{F} = (\mathfrak{R}, G, \alpha)$ consists of a von Neumann algebra $\mathfrak{R}$ (on a Hilbert space $\mathcal{H}$), a locally compact abelian group G, and a homomorphism $g \to \alpha_g : G \to \mathrm{Aut}(\mathfrak{R})$ (where $\mathrm{Aut}(\mathfrak{R})$ is the group of *-automorphisms of $\mathfrak{R}$) which is continuous when $\mathrm{Aut}(\mathfrak{R})$ is endowed with the topology of pointwise σ-weak ($\equiv$ ultraweak) convergence. $\mathfrak{F}$ (or α) is said to be *inner* if there is a U in $\mathrm{Rep}(G, \mathcal{H})$, the set of all strongly continuous unitary representations of G on $\mathcal{H}$, such that U implements α (i.e., $\alpha_g(A) = U_g A U_g^*$ for all g in G and A in $\mathfrak{R}$), and $U_g \in \mathfrak{R}$ for all $g \in G$. In the Haag-Kastler formulation of quantum field theory [11], the algebra of local observables for a given region of space-time is a von Neumann algebra $\mathfrak{R}$, and the time evolution of the system is given by a one-parameter dynamical system $(\mathfrak{R}, \mathsf{R}, \alpha)$. Moreover, the symmetries of the system are also given by *-automorphisms of $\mathfrak{R}$, so every locally compact (abelian) group of symmetries of the system gives rise to a dynamical system. If such a dynamical system is inner, then this can be interpreted to mean that the symmetries are observable.

In 1966, Borchers showed [2] that if $\mathfrak{F} = (\mathfrak{R}, \mathsf{R}, \alpha)$ is a one-parameter dynamical system, and if α is implemented by a $U \in \mathrm{Rep}(G, \mathcal{H})$ with $\mathrm{sp}\, U \subset [0, \infty)$ (corresponding to positive energy), then $\mathfrak{F}$ is inner (so the dynamics of the system are observable). This is a satisfying result from a physical standpoint, and also an intriguing result from a mathematical standpoint, since the crucial assumption here is that the spectrum of U is positive. Without this assumption, the result is false. In fact, if $(\mathfrak{R}, G, \alpha)$ is *any*

Received April 9, 1979.